J. Math. Soc. Japan
Vol. 38, No. 4, 1986

Real analytic actions of complex symplectic groups and other classical Lie groups on spheres

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(Received April 9, 1985)

0. Introduction.

There seems to be few works on non-compact semi-simple Lie groups acting on the sphere non-transitively. In the previous papers [7], [8] we have studied analytic $\boldsymbol{SL}(n, \boldsymbol{R})$ (resp. $\boldsymbol{SL}(n, \boldsymbol{C})$) actions on the standard k-sphere and we have shown that such an action has been characterized by an analytic $\boldsymbol{R}_0$ (resp. $\boldsymbol{C}_0$) action on a homotopy $(k-n+1)$-sphere (resp. $(k-2n+2)$-sphere) satisfying a certain condition for $5\leqq n\leqq k\leqq 2n-2$ (resp. $n\geqq 7$ and $2n\leqq k\leqq 4n-2$). Here $\boldsymbol{R}_0$ (resp. $\boldsymbol{C}_0$) denotes the multiplicative group of all non-zero real (resp. complex) numbers.

In this paper we study analytic $\boldsymbol{Sp}(n, \boldsymbol{C})$ actions on integral homology k-spheres and we shall show in Section 5 that such an action is characterized by an analytic $\boldsymbol{C}_0$ action on an integral homology $(k-4n+2)$-sphere satisfying a certain condition for $n\geqq 7$ and $4n\leqq k\leqq 8n-2$. By an integral homology k-sphere we mean a closed orientable analytic manifold whose homology with integer coefficients is isomorphic to that of the standard k-sphere.

Our method and result are quite similar to that of the previous papers [7], [8]. One difference here is the need to show that the fixed point set of the restricted $L(n)$ action is an analytic submanifold of a given manifold with certain analytic $\boldsymbol{Sp}(n, \boldsymbol{C})$ action, where $L(n)$ is a non-compact closed subgroup of $\boldsymbol{Sp}(n, \boldsymbol{C})$ defined in Section 1. To show it, we need to study certain analytic $\boldsymbol{SL}(2, \boldsymbol{C})$ actions. Theorem 2.1 is a key result.

In the final part of Section 5, we describe transitive $\boldsymbol{Sp}(n, \boldsymbol{C})$ actions on $(4n-1)$-sphere. Finally, we study analytic $\boldsymbol{SO}(n, \boldsymbol{C})$ actions on $(2n-1)$-sphere and on the Brieskorn variety $W^{2n-1}(d)$, and analytic $\boldsymbol{SL}(n, \boldsymbol{R})$ actions on $(2n-1)$-sphere in Section 6.

1. Certain closed subgroups of $\boldsymbol{Sp}(n, \boldsymbol{C})$.

1.1. Let $\boldsymbol{GL}(m, \boldsymbol{C})$ and $\boldsymbol{U}(m)$ denote the group of regular matrices of degree m with complex coefficients and the group of unitary matrices of degree m,