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On removability of sets for holomorphic and harmonic functions

Dedicated to Professor Yukio Kusunoki on his sixtieth birthday

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1. Introduction.

Let W be an open set in the complex plane $\boldsymbol{C}$. For a function f on W, denote by $S(f)$ the set of all points at which f fails to admit a complex derivative; as noted in Kaufman [**4**], $S(f)$ is a Borel subset of W if f is a Borel measurable function on W.

We say that a function h on the interval $[0, \infty)$ is a measure function if $h(0)=0$, $h(r)>0$ for $r>0$, h is nondecreasing on $[0, \infty)$ and further

$$h(2r) \leqq \text{const.}\ h(r) \qquad \text{for } r>0$$

(cf. Carleson [**2**]). We denote by Λ_h the Hausdorff measure associated with the measure function h, which is defined by

$$\Lambda_h(E) = \lim_{\delta \downarrow 0} \inf \left\{ \sum_{j=1}^{\infty} h(r_j)\ ;\ r_j \leqq \delta,\ \bigcup_{j=1}^{\infty} B(z_j, r_j) \supset E \right\}$$

for a set E, where $B(z, r)$ denotes the open disc with center at z and radius r. If $h(r)=r^{\alpha}$, $\alpha>0$, then we shall write Λ_α for Λ_h.

Let $1 \leqq p \leqq \infty$ and $1/p+1/p'=1$. For a measure function h and a locally integrable (Borel) function f on W, define

$$F(z) = \sup_{B} r^{-1-2/p} h(r)^{-1/p'} \inf_{g} \int_B |f(w)-g(w)|\, d\Lambda_2(w)\,,$$

where the supremum is taken over all open discs B with radius r such that $z \in B \subset W$ and the infimum is taken over all functions g which is holomorphic in B.

Our first aim is to establish the following result.

THEOREM 1. *Suppose* $F \in L^p(W)$.

(i) *If* $p<\infty$, $\lim_{r \downarrow 0} r^{-2} h(r) = \infty$ *and* $\Lambda_h(S(f))<\infty$, *then* f *can be corrected on a set of measure zero to be holomorphic in* W.