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Gap theorems for minimal submanifolds of Euclidean space

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0. Introduction.

The purpose of the present paper is to prove the following

THEOREM A. *Let M be a connected, complete minimal submanifold properly immersed into Euclidean space* $\boldsymbol{R}^N$. *Suppose that*

$$\text{(0.1)} \qquad \textit{the scalar curvature of } M \textit{ at } x \geqq -\frac{A}{1+|x|^{2+\varepsilon}}$$

for some positive constants A and ε, where $|x|$ stands for the Euclidean norm of $x \in M \subset \boldsymbol{R}^N$. *Then:*

(I) *M is an m-plane if $m=\dim M \geqq 3$ and M has one end, or if $m=2$, $\varepsilon \geqq 2$ and M has one end.*

(II) *M is a hyperplane if $m=N-1$, $2+\varepsilon>2m$ and M is embedded into* $\boldsymbol{R}^N$.

(III) *M is a catenoid if $m \geqq 3$, $m=N-1$ and M has two ends, or if $m=2$, $N=3$ and M has two embedded ends.*

Since an area-minimizing hypersurface properly embedded into $\boldsymbol{R}^N$ has one end (cf. [**1**]), we have the following

COROLLARY 1. *Let M be an area-minimizing hypersurface properly embedded into $\boldsymbol{R}^N$ satisfying condition* (0.1). *Then M is a hyperplane of* $\boldsymbol{R}^N$.

In case M is a complex submanifold properly embedded into $\boldsymbol{C}^N$, condition (1.0) will imply that the volume of the exterior metric ball $M \cap B_e(r)$ with radius r grows like r^{2m} ($m=\dim_{\boldsymbol{C}} M$) (cf. Lemma 2 (1)), and hence by a theorem of Stoll [**16**], M turns out to be algebraic. In particular, M has one end if $m \geqq 2$ (cf. Lefschetz hyperplane theorem). Thus we have proven

COROLLARY 2. *Let M be a complex submanifold properly embedded into* $\boldsymbol{C}^N$.

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