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Rough isometries and the parabolicity of riemannian manifolds

By Masahiko KANAI

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1. Introduction.

When we study non-compact complete riemannian manifolds, we observe that quasi-isometric deformations of their metrics do not alter global or qualitative properties of the manifolds: For example, two complete riemannian manifolds quasi-isometric to each other obviously have the same volume growth rate. On the other hand, for a non-compact complete riemannian manifold, "attaching finitely many handles" (see Fig. 1) also preserves such geometric invariants of the manifold; in other words, we may say that a local topological deformation of the manifold does not exert essential influences on global geometry. Suggested by these observations, we introduced the notion of rough isometry in [**10**]. A map $\varphi: X \to Y$, not necessarily continuous, between two metric spaces X and Y is called a *rough isometry* if the following two conditions are satisfied:
(i) for some $\varepsilon > 0$, the ε-neighborhood of the image of φ in Y covers Y;
(ii) there are constants $a \geqq 1$ and $b \geqq 0$ such that

$$a^{-1} d(x_1, x_2) - b \leqq d(\varphi(x_1), \varphi(x_2)) \leqq a d(x_1, x_2) + b \qquad \text{for all} \quad x_1, x_2 \in X.$$

A metric space X is said to be *roughly isometric* to a metric space Y if there exists a rough isometry from X into Y. Evidently being roughly isometric is an equivalence relation among metric spaces. Also, since we do not impose continuity to rough isometries, there are a lot of pairs of complete riemannian manifolds which are roughly isometric to each other but are not homeomorphic; e.g., two manifolds in Fig. 1. Nevertheless, some geometric attributes of riemannian manifolds are inherited through rough isometries. In fact we proved the

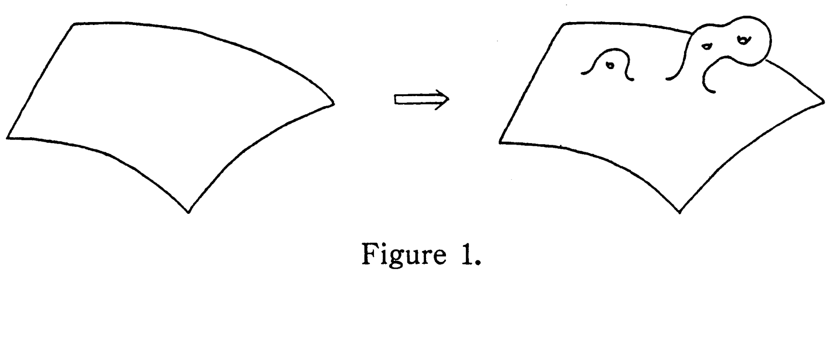

Figure 1.