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Inverse problems for heat equations on compact intervals and on circles, I

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§ 1. Introduction.

The purpose of the present paper is to study uniqueness of certain inverse problems for heat equations.

For $p\in C^1[0, 1]$, $h\in \boldsymbol{R}$, $H\in \boldsymbol{R}$ and $a\in L^2(0, 1)$, let $(E_{p,h,H,a})$ be the heat equation

$$\frac{\partial u}{\partial t}+\left(p(x)-\frac{\partial^2}{\partial x^2}\right)u=0 \qquad (0<t<\infty\,;0<x<1) \tag{1.1}$$

with the boundary condition

$$\left(\frac{\partial u}{\partial x}-hu\right)\Big|_{x=0}=\left(\frac{\partial u}{\partial x}+Hu\right)\Big|_{x=1}=0 \qquad (0<t<\infty) \tag{1.2}$$

and with the initial condition

$$u|_{t=0}=a(x) \qquad (0<x<1). \tag{1.3}$$

As is known, the solution $u=u(t, x)$ exists uniquely for given coefficients and initial value (p, h, H, a). However, let these (p, h, H, a) be unknown, and instead the values $u(t, 0)$ and $u(t, x_0)$ be observed for $t\in[T_1, T_2]$ and $x_0\in(0, 1]$, where $0\leqq T_1<T_2<\infty$. Do the data $\{u(t, 0), u(t, x_0)\,|\,T_1\leqq t\leqq T_2\}$ determine (p, h, H, a)? This kind of problem is called an inverse problem, and is formulated more precisely as follows.

Consider the mapping

$$F^1=F^1_{T_1,T_2,x_0}:\ (q, j, J, b)\longmapsto\{v(t, 0), v(t, x_0)\,|\,T_1\leqq t\leqq T_2\}\,, \tag{1.4.1}$$

where $v=v(t, x)$ is the solution of $(E_{q,j,J,b})$. Let $(p, h, H, a)\in C^1[0, 1]\times\boldsymbol{R}\times\boldsymbol{R}\times L^2(0, 1)$ be given and $u=u(t, x)$ be the solution of $(E_{p,h,H,a})$. Then the set

$$\boldsymbol{M}^1_{p,h,H,a,x_0}\equiv(F^1_{T_1,T_2,x_0})^{-1}(F^1_{T_1,T_2,x_0}(p, h, H, a)) \tag{1.5.1}$$

denotes the totality of equations $(E_{q,j,J,b})$ whose solutions have the same values as those of u on $\xi=0, x_0$. Namely,