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Meromorphic solutions of some nonlinear difference equations of higher order

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1. Introduction.

Here we will consider the nonlinear difference equation

$$\alpha_n y(x+n)+\alpha_{n-1}y(x+n-1)+\cdots+\alpha_1 y(x+1)=R(y(x)), \tag{1.1}$$

where $R(y)$ is a rational function of y:

$$\begin{cases} R(y)=P(y)/Q(y), \\ P(y)=a_p y^p+\cdots+a_0, \\ Q(y)=b_q y^q+\cdots+b_0, \end{cases} \tag{1.2}$$

in which $\alpha_n, \cdots, \alpha_1$; $a_p, \cdots, a_0$; $b_q, \cdots, b_0$ are constants, $\alpha_n a_p b_q \neq 0$. We suppose that $P(y)$ and $Q(y)$ are mutually prime. In the sequel, we denote by p and q the degree of the nominator $P(y)$ and of the denominator $Q(y)$, respectively.

We will investigate in this note whether the equation (1.1) admits a meromorphic solution or not. Of course, we mean nontrivial solution, i.e., solution which is not identically equal to a constant.

In [1] and [2], Harris and Sibuya investigated the difference equation

$$\begin{aligned} &\vec{y}(x+1)=\vec{F}(x, \vec{y}(x)), \\ &\vec{F}(x, \vec{y})=(F_j(x, y_1, \cdots, y_n), \quad j=1, \cdots, n), \\ &\vec{F}(\infty, \vec{0})=\vec{0}. \end{aligned} \tag{1.3}$$

When F_j are rational functions of $x, y_1, \cdots, y_n$, then their results imply that the equation (1.3) possesses a meromorphic solution $\vec{y}(x)$ which has an asymptotic expansion

$$\vec{y}(x) \sim \sum_{m=1}^{\infty} \vec{a}_m / x^m \tag{1.4}$$

in an angular domain. This is a very general result. But in the present case (1.1), the solution (1.4) obtained by them has coefficients $\vec{a}_m=\vec{0}$, $m=1, 2, \cdots$. Therefore we need somewhat more detailed study of the equation to get nontrivial solutions.

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