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On the spaces of self homotopy equivalences of certain CW complexes

Dedicated to Professor Nobuo Shimada on his 60th birthday

By Tsuneyo YAMANOSHITA

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§ 0. Introduction.

Let X be a connected locally finite CW complex with non-degenerate base point and let $G(X)$ and $G_0(X)$ be the spaces of self homotopy equivalences of X and self homotopy equivalences of X preserving the base point respectively.

It seems that little is known about the homotopy type of $G(X)$ except in the following two cases. When X is an Eilenberg-MacLane complex $K(\pi, n)$, the weak homotopy type of $G(X)$ is determined completely. That is, Thom noted that if π is an abelian group $G(K(\pi, n))$ has the same weak homotopy type as $\mathrm{Aut}(\pi)\times K(\pi, n)$, where $\mathrm{Aut}(\pi)$ denotes the group of automorphisms of π [**7**]. Gottlieb proved that $G(K(\pi, 1))$ has the same weak homotopy type as $\mathrm{Out}(\pi)\times K(Z(\pi), 1)$, where $\mathrm{Out}(\pi)$ denotes the group of automorphisms of π modulo the inner automorphisms and $Z(\pi)$ denotes the center of π [**1**]. When X is the n-sphere $S^n\,(n\geqq 1)$, it is known that $\pi_i(G_0(S^n))\cong\pi_{n+i}(S^n)$ $(i\geqq 1)$.

In this paper, we shall show the following two theorems and their applications.

THEOREM A. *Let X and Y be connected locally finite CW complexes with base points. For a given $n>0$, assume that $\pi_i(X)=0$ for every $i>n$ and $\pi_i(Y)=0$ for every $i\leqq n$. Then we have*

$$G(X\times Y)=G(X)^Y\times G(Y)^X,$$

$$G_0(X\times Y)=(G(X),\, G_0(X))^{(Y,\, y_0)}\times(G(Y),\, G_0(Y))^{(X,\, x_0)},$$

where $(Z, Z')^{(K,L)}$ denotes the space of maps of (K, L) into (Z, Z').

THEOREM B. *Let X be a connected locally finite CW complex with base point whose dimension is not greater than n and let Y be an n-connected locally finite CW complex with base point. Then the same formulas as in Theorem A hold for $G(X\times Y)$ and $G_0(X\times Y)$.*