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Maximal surfaces with conelike singularities

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A spacelike surface in the 3-dimensional Minkowski space $L^3=(\boldsymbol{R}^3, dx^2+dy^2-dz^2)$ is said to be *maximal* if the mean curvature vanishes identically. Any spacelike surface in L^3 can be represented locally as the graph $\{z=u(x, y)\}$ of a smooth function u with $u_x^2+u_y^2<1$. Then the surface is maximal if u satisfies the equation:

$$(1-u_x^2)u_{yy}+2u_xu_yu_{xy}+(1-u_y^2)u_{xx}=0.$$

This equation is elliptic when $u_x^2+u_y^2<1$, but the ellipticity degenerates when $u_x^2+u_y^2$ tends to 1. Related to this fact, maximal surfaces often have singularities which are of different kinds from those appearing in minimal surfaces in the Euclidean space. For example, consider a surface S in L^3 defined by

$$\begin{cases} x(\rho, \theta)=\dfrac{1}{k+1}\sinh((k+1)\rho)\cos((k+1)\theta)+\dfrac{1}{k-1}\sinh((k-1)\rho)\cos((k-1)\theta), \\ \qquad \left(=\dfrac{1}{2}\sinh 2\rho\cos 2\theta+\rho, \text{ if } k=1\right), \\ y(\rho, \theta)=\dfrac{1}{k+1}\sinh((k+1)\rho)\sin((k+1)\theta)-\dfrac{1}{k-1}\sinh((k-1)\rho)\sin((k-1)\theta), \\ \qquad \left(=\dfrac{1}{2}\sinh 2\rho\sin 2\theta, \text{ if } k=1\right), \\ z(\rho, \theta)=-\dfrac{2}{k}\sinh(k\rho)\cos(k\theta), \quad (=-2\rho, \text{ if } k=0), \quad \rho>0, \quad 0\leqq\theta<2\pi, \end{cases}$$

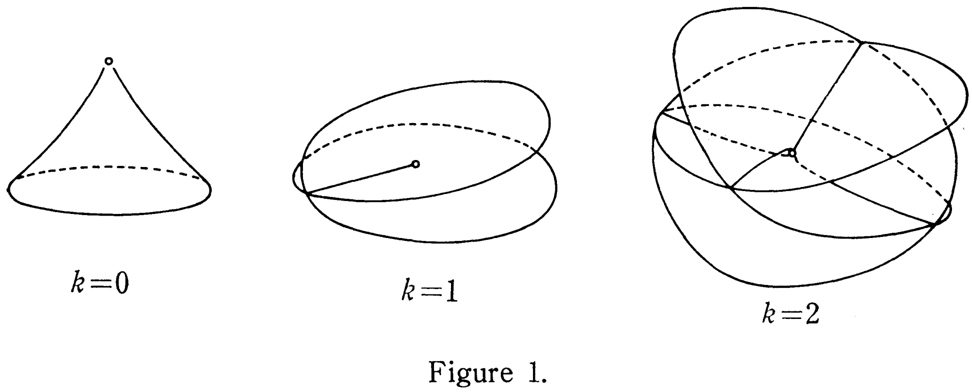

Figure 1.