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A classification of type I AW^*-algebras and Boolean valued analysis

By Masanao OZAWA

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1. Introduction.

The aim of this paper is to give a complete classification of type I AW^*-algebras using Boolean valued analysis. The structure theory of type I AW^*-algebras was instituted by Kaplansky [6] as a purely algebraic generalization of type I von Neumann algebras. The structure theory of type I von Neumann algebras leads an essentially unique direct sum decomposition into homogeneous von Neumann algebras. Thus a complete system of $*$-isomorphism invariants for such an algebra is obtained as a set of cardinals together with partition of unity consisting of central projections up to automorphism of the center. Kaplansky's theory of type I AW^*-algebras succeeded in decomposing every type I AW^*-algebra into homogeneous AW^*-algebras, but his theory was not completed as he stated [**6**; p. 460], "One detail has resisted complete solution thus far: the uniqueness of the cardinal number attached to a homogeneous AW^*-algebra of type I."

In this paper, we shall show that the solution of the above cardinal uniqueness problem is negative, as conjectured by Kaplansky [**7**; p. 843, footnote]. This means that we cannot insure the uniqueness of the direct sum decomposition of type I AW^*-algebras into homogeneous algebras. Thus the structure of $*$-isomorphism invariants for type I AW^*-algebras is supposed to be more complicated. However, as we shall show in this paper, it is a surprising fact that we can find $*$-isomorphism invariants for such algebras in the objects already studied in the field of mathematical logic. Precisely, we shall show that cardinal numbers in Scott-Solovay's Boolean valued universe of sets constitute $*$-isomorphism invariants of type I AW^*-algebras.

Boolean valued analysis is our method which bridges the gap between the results of mathematical logic and the problems of analysis. This new method of analysis was introduced by D. Scott and R. Solovay when they reformulated the theory of P. J. Cohen's forcing in terms of Boolean valued models of set theory in 1966. Recently, Boolean valued analysis was developed by G. Takeuti in operator theory, harmonic analysis and operator algebras ([**12**], [**13**], [**14**],