A construction of geometric structures on Seifert fibered spaces

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§ 1. Introduction.

Since Thurston described his geometric views on 3-manifold topology, we have had quite many opportunities of recognizing its incredible importance. According to this recognition, a couple of expository descriptions of geometric structures on 3-manifolds, say in [4] [6], have been presented. In particular in the article by Peter Scott [6] there is a complete description of geometric structures on Seifert fibered spaces. A geometric structure on a Seifert fibered space there is given by finding a faithful discrete representation of its fundamental group to the isometric transformation group. In this paper we shall give an alternative construction of geometric structures by using the idea of Thurston's hyperbolic Dehn surgery. We shall describe this only for Seifert fibered spaces over S^2 with precisely 3 exceptional fibers since it is the typical case and simultaneously the method is easily generalized to the other case.

An immediate application of the construction is computation of volumes. The normalized volume for a closed Lorentz manifold is known to be a topological invariant (see [2], [4]). We shall take the trefoil knot as an example for computing volumes of the resultant manifolds of Dehn surgery along it. Though taking the trefoil seems to be special for general discussion, it is enough to see how the volumes fill up R^+, which is in contrast with the case of hyperbolic volumes.

In the next three sections, we discuss geometric Dehn surgery in general situation and review 3-dimensional geometries in the sense of Thurston [8] and Seifert fibrations. The main construction and Dehn surgery along the trefoil knot are dealt in the last two sections.

§ 2. Geometric Dehn surgery.

In this section, we generalize Thurston's hyperbolic Dehn surgery [7]. Roughly speaking, it is the study of completion of an incomplete geometric toral

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