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On the behavior at infinity of logarithmic potentials

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1. Statement of results.

For a (signed) measure λ in the plane R^2, we define

$$L\lambda(x)=\int \log\frac{1}{|x-y|}d\lambda(y)$$

if the integral exists at x. We note that $L\lambda(x)$ is finite for some x if and only if

$$\int \log(1+|y|)d|\lambda|(y)<\infty, \tag{1}$$

where $|\lambda|$ denotes the total variation of λ. Denote by $B(x, r)$ the open disc with center at x and radius r. For $E\subset B(0, 2)$ we set

$$C(E)=\inf \mu(R^2),$$

where the infimum is taken over all nonnegative measures μ on R^2 such that S_μ (the support of μ)$\subset B(0, 4)$ and

$$\int \log\frac{8}{|x-y|}d\mu(y)\geqq 1 \qquad \text{for every} \quad x\in E.$$

A set E in R^2 is said to be thin at infinity if

$$\sum_{j=1}^{\infty} jC(E_j')<\infty, \quad E_j'=\{x\in B(0, 2)-B(0, 1)\,;\, 2^j x\in E\}. \tag{2}$$

It is known (cf. Brelot [1; Theorem IX, 7]) that if μ is a nonnegative measure on R^2 satisfying (1), then there exists a set $E\subset R^2$ which is thin at infinity and for which

$$\lim_{|x|\to\infty,\, x\in R^2-E}[L\mu(x)+\mu(R^2)\log|x|]=0.$$

Our first aim is to establish the following result.

THEOREM 1. *Let μ be a nonnegative measure on R^2 satisfying* (1). *Then there exists a set E in R^2 such that*

$$\lim_{|x|\to\infty,\, x\in R^2-E}(\log|x|)^{-1}L\mu(x)=-\mu(R^2),$$

$$\sum_{j=1}^{\infty} j^2C(E_j')<\infty, \tag{3}$$