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Pseudo-differential operators and Markov processes

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0. Introduction.

Given a Lévy type generator L acting on test functions on $\boldsymbol{R}^d$, there are various formulations of Markov processes associated with L. One is a weak solution of the stochastic differential equation of jump type with coefficients corresponding to the local characteristics of the operator L. Another is a Markov process whose resolvent $\{R_\lambda\}$ satisfies that $R_\lambda(\lambda-L)f=f$ for any test function f. These formulations are unified as the martingale problem for the operator L. Each probability measure P on the path space is said to solve the martingale problem for L if the process

$$f(X_t)-f(X_0)-\int_0^t Lf(X_s)ds$$

is a P-martingale for any test function f on $\boldsymbol{R}^d$. The martingale problem was introduced by Stroock and Varadhan [**15**] to prove the uniqueness of the diffusion process whose generator is a given elliptic differential operator with continuous coefficients. In the present paper we shall discuss the existence and the uniqueness of solutions of the martingale problem for a class of *non-degenerate* Lévy type generators L whose local characteristics are not always continuous, to prove the existence and the uniqueness of Markov processes with jumps having L as their generators. Grigelionis [**6**] and [**7**] gave another martingale formulation for jump type processes.

We shall say that a Lévy type generator L is non-degenerate if it is so as a pseudo-differential operator, i.e. there is a constant α, $0<\alpha\leqq 2$, and

$$e^{-ix\cdot\xi}L(e^{ix'\cdot\xi})=\psi^{(\alpha)}(x, \xi)+\phi^{(\alpha)}(x, \xi),$$

where $\psi^{(\alpha)}(x, \xi)$ is a homogeneous function in ξ with index α such that the real part of $\psi^{(\alpha)}(x, \xi)$ is strictly negative for $\xi\neq 0$, and $\phi^{(\alpha)}(x, \xi)=o(|\xi|^\alpha)$ for large $|\xi|$. In the case $\alpha=2$, the existence and the uniqueness were discussed by Komatsu [**9**] and Stroock [**14**]. So far, for $\alpha\neq 2$, they have been investigated only in the context that the real part of the principal part $\psi^{(\alpha)}(x, \xi)$ of the symbol of L is independent of the variable x. Tsuchiya [**17**] investigated the