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C^*-algebras associated with shift dynamical systems

Dedicated to the memory of Professor Teishirô Saitô

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Introduction.

In this paper, we study the structure of C^*-algebras generated by shift operators on a separable Hilbert space $\mathfrak{H}$ associated with a fixed basis $\{e_n\}_{n\in Z}$ where Z is the set of all integers. A pair (Ω, σ) is said to be a topological dynamical system if Ω is a compact (Hausdorff) space and σ is a homeomorphism of Ω. According to O'Donovan [**6**], our C^*-algebras correspond to the class of topological dynamical systems which satisfy the condition: there exists a map ϕ of Z onto a dense subset of Ω such that $\sigma(\phi(n))=\phi(n+1)$ for each n in Z. We call each of these systems a shift dynamical system and denote by (Ω, σ, ϕ). Although O'Donovan studied mainly the C^*-algebras generated by a weighted shift, we examine ones generated by a family of shift operators.

Recently, by Rieffel [**15**] and Pimsner-Voiculescu [**13**], the irrational rotation C^*-algebras were completely classified by using the K_0-groups. Furthermore, Riedel [**14**] generalized their work to the C^*-algebras associated with minimal rotations on the dual groups of countable discrete subgroups of the one-dimensional torus $T=\{z\in C; |z|=1\}$ where C is the set of all complex numbers. Each of these C^*-algebras associated with minimal rotation can be considered as a C^*-algebra generated by shift operators in our sense.

In Section 1, we discuss some general properties and two kinds of conjugacies of shift dynamical systems, and consider the fundamental properties (e.g. simplicity, existence of tracial state) of C^*-algebras associated with those systems. In Section 2, we show that the structure of simple C^*-algebras corresponding to the discrete subgroups G (not necessarily countable subgroup) of T is completely determined by G. This generalizes Riedel's results. Furthermore we give a necessary and sufficient condition for a shift dynamical system to be one associated with a discrete subgroup of T. In Section 3, we discuss the case where ϕ is a homeomorphism of Z onto the subspace $\phi(Z)$ of Ω. This is equivalent

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