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A singular quasilinear diffusion equation in L^1

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1. Introduction.

The initial and Dirichlet boundary value problem for the equation $u_t-\Delta\alpha(u)+\beta(u)\ni f$ has a unique generalized or integral solution in L^1 when α and β are maximal monotone graphs in $\boldsymbol{R}$, each containing the origin, and at each point of their common domain either one of α or β is single-valued. Weak Maximum and Comparison Principles follow from an L^∞ estimate on the solution and from an L^1 estimate on the difference of solutions, respectively. This L^1 integral solution is shown to satisfy the above partial differential equation in the sense of distributions when α is surjective (or the data is bounded) and β is continuous.

We shall consider the initial-boundary-value problem

(1.a) $u_t-\Delta v+w=f,\ v\in\alpha(u),\ w\in\beta(u)$ in Ω

(1.b) $v=0$ on $\partial G\times(0, T)$

(1.c) $u=u_0$ on $G\times\{0\}$

where G is a bounded domain in $\boldsymbol{R}^n$, $\Omega\equiv G\times(0, T)$, Δ is the Laplacian in $\boldsymbol{R}^n$, and α and β are maximal monotone graphs in $\boldsymbol{R}\times\boldsymbol{R}$, each containing the origin. The problem (1) will be regarded as an abstract Cauchy problem of the form

(2.a) $u'(t)+A(u(t))+B(u(t))\ni f(t)$, a.e. $t\in(0, T)$

(2.b) $u(0)=u_0$

in the Banach space $L^1(G)$. An *integral solution* of (2) in a Banach space X is a $u\in C(0, T; X)$ such that $u(0)=u_0$ and $u(t)\in\overline{\mathrm{dom}(A+B)}$,

$$\frac{1}{2}\|u(t)-x\|^2\leqq\frac{1}{2}\|u(s)-x\|^2+\int_s^t\langle f(\tau)-y,\ u(\tau)-x\rangle d\tau$$

for each $y\in(A+B)(x)$ and $0\leqq s\leqq t\leqq T$. The pairing in the integral is the semi-scalar-product

$$\langle y,\ x\rangle\equiv\sup\{(y,\ x^*): x^*\in X^*,\ x^*(x)=\|x\|=\|x^*\|\}$$

on the Banach space X. A multi-valued operator $A\subset X\times X$ is called *accretive* if