J. Math. Soc. Japan
Vol. 35, No. 4, 1983

Semi-simple degree of symmetry and maps of degree one into a product of 2-spheres

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(Received June 5, 1982)
(Revised Oct. 4, 1982)

Introduction.

Recently many authors have shown that if a smooth closed manifold M admits a continuous map of degree one into a product of 1-spheres, then the compact connected Lie group which acts on M smoothly and almost effectively is a torus ([**5**], [**9**], [**11**]). The present note is motivated by this result. In this note, we shall study the semi-simple degree of symmetry of a manifold which admits a continuous map of degree one into a product of 2-spheres. Here the *semi-simple degree of symmetry* of a manifold is, by definition, the maximum dimension of the compact connected semi-simple Lie group which acts on the manifold smoothly and almost effectively.

We shall prove the following

THEOREM A. *Let M be a simply connected closed $2m$-dimensional topological manifold which admits a continuous map of degree one into a product of 2-spheres. Then $SO(3)$ or $SU(2)$ is only the compact connected simple Lie group which acts on M continuously and almost effectively. Therefore if a compact connected Lie group G acts on M continuously and almost effectively, then G is locally isomorphic to $T^s\times SU(2)\times \cdots \times SU(2)$.*

A typical example of M of Theorem A is a connected sum of $S^2\times \cdots \times S^2$ (m-times) and a $2m$-dimensional manifold. As for the connected sum, we shall obtain the following

THEOREM B. *Let M be as in Theorem* A *and N a simply connected closed $2m$-dimensional topological manifold which is not a rational homology sphere. Then the connected sum $X=M\# N$ does not admit any action of $SU(2)$.*

REMARK 1. As a corollary to Theorem B, we have the following

PROPOSITION. *Let N be as in Theorem* B. *Then the semi-simple degree of symmetry of $(S^2\times \cdots \times S^2)\# N$ is zero.*

REMARK 2. Since the connected sum $M=(S^2\times \cdots \times S^2)\#\Sigma^{2m}$ (Σ^{2m} : $2m$-dimensional homotopy sphere) is homeomorphic to $S^2\times \cdots \times S^2$, it admits a continuous action of $SU(2)$. But it does not necessarily admit a smooth action of $SU(2)$.