Some results on weakly normal ring extensions

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The purpose of this paper is to give some results on weakly normal ring extensions which correspond to those on seminormal ring extensions obtained by several authors in [4], [7], [10], [12] and [13]. In the paper [8] M. Manresi gave a new characterization of weak normalization and discussed some questions related closely to our results. But our method depends on a criterion for weak normality which was given by S. Itoh in [6] and corresponds to Hamann's criterion for seminormality given in [5] and [7].

In §1 we shall give a simple proof of this criterion different from the proof given in [6] for convenience' sake and also give a new characterization of weak normalization in the case where the characteristic of rings is a positive prime number. In §2 we show first faithfully flat descent of weak normality which is a special case of pull-back descent of the property. Then we discuss local properties of weak normality. Furthermore we give some conditions for faithfully flat ring extensions to be weakly normal. These results are all given without any noetherian hypothesis. In the last section we generalize the notions of glueings of prime ideals or primary ideals which were defined in [12] and [11], and we show some basic results on these generalized glueings. In particular we give the notion of a weak glueing of a ring which plays a role for weakly normal ring extensions similar to the one played by ordinary glueings of prime ideals for seminormal ring extensions and show a structure theorem for weak normal ring extensions of noetherian rings corresponding to Theorem 2.1 in [12] for seminormal cases. Lastly we show results related to the going-down of Serre's property (S_2) under generalized glueings of rings corresponding to Theorem 2 and its Corollary in [13].

All the rings in this paper are commutative with unit and we use freely the terminology and results in [9].