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Fluctuations of Markovian systems in Kac's caricature of a Maxwellian gas

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0. Introduction.

Kac [**5**] exhibited with his caricature of a Maxwellian gas, that the spatially homogeneous solution of Boltzmann problem is obtained as a limit of empirical distributions induced from n molecules Markov processes (as $n\to\infty$) which are regulated by master equations associated with the collision operator in the Boltzmann equation. Kac [**6**] also considered a fluctuation problem: he gave a formal derivation for a convergence of fluctuations of the empirical distributions about the solution of Boltzmann problem and observed that a kind of Ornstein-Uhlenbeck process appears in the limit. To this problem McKean [**8**] gave a rigorous result for his model of a two speed Maxwellian gas and made also a heuristic argument for the model of a gas of hard balls. Recently H. Tanaka [**11**] treated the same problem for Kac's caricature and obtained a convergence result in an equilibrium case.

In this paper we shall study the fluctuation problem for Kac's caricature and prove, in nonequilibrium (as well as equilibrium) cases, that the family of distributions on $D[[0, \infty), \mathcal{S}_\delta']$ induced by fluctuation processes converges weakly to a distribution of a kind of time-inhomogeneous Ornstein-Uhlenbeck process on $\mathcal{S}_\delta'$, where $\mathcal{S}_\delta'$ is a Hilbert space of tempered distributions.

To get the convergence result we shall follow the martingale approach as exposed in Stroock-Varadhan's book [**10**] and as applied by Holley-Stroock [**4**] to handle a convergence in law of tempered distribution valued Markov processes. Guided by their schedule, we shall first prove the tightness of the fluctuation processes as $\mathcal{S}_\delta'$-valued processes (§ 4), then that any limiting law solves an associated martingale problem (§ 5), and finally a uniqueness of a solution of the martingale problem (§ 6). A similar approach to the present problem has been already adopted by H. Tanaka [**11**].

In § 1 we shall review the Kac's model and his result. In § 2 we shall in-

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