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A characterization of the hermitian quadrics

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Introduction.

Let M be a $(2n-1)$-dimensional manifold and S a subbundle of the complexified tangent bundle $\boldsymbol{C}T(M)$ of M. Then S is called a PC (or CR) structure if it satisfies the following conditions: 1) $S\cap\bar{S}=\{0\}$, 2) $[\Gamma(S), \Gamma(S)]\subset\Gamma(S)$ and 3) $\dim_{\boldsymbol{C}} S=n-1$. The manifold M equipped with the PC structure S is called a PC (or CR) manifold. Furthermore following Tanaka [**5**], we say that a PC manifold M is normal if there is given an infinitesimal automorphism ξ which is transversal to the subbundle $S+\bar{S}\subset\boldsymbol{C}T(M)$.

For example consider the hermitian quadric Q_r of the n-dimensional complex projective space $P_n(\boldsymbol{C})$ defined by the equation

$$\sum_{j=0}^{r}|z_j|^2-\sum_{k=r+1}^{n}|z_k|^2=0\,,$$

where $0\leqq r\leqq\dfrac{n-1}{2}$. For a given positive number c, let $\xi_{(c)}$ be the vector field on Q_r induced from the 1-parameter group of transformations

$$\tau_t([z_0, \cdots, z_n])=[z_0, \cdots, z_r, e^{\sqrt{-1}ct}z_{r+1}, \cdots, e^{\sqrt{-1}ct}z_n]\,,$$

where $[z_0, \cdots, z_n]\in Q_r$. Then we can see that Q_r is endowed with a PC structure and that the PC manifold Q_r is normal with respect to the vector field $\xi_{(c)}$.

The main purpose of the present paper is to characterize the hermitian quadrics in terms of normal PC structures.

Let us now proceed to the description of the main results in the present paper. Let (M, ξ) be a normal PC manifold. We assume that M is compact and non-degenerate of index r and that (M, ξ) satisfies Condition (C) (This condition requires that the PC structure S is suitably decomposed into subbundles S^1 and S^2. For the details, see § 1). Then we know that to the normal PC manifold (M, ξ) together with the decomposition of S, there are naturally associated a Riemannian metric g, the canonical affine connection ∇, and two kinds of scalar curvatures σ_1 and σ_2. Furthermore let $\mathfrak{a}(M)$ be the Lie algebra of all infinitesimal automorphisms of the PC manifold M, and let $\mathfrak{c}(M)$ be the centralizer of ξ in $\mathfrak{a}(M)$, which is nothing but the Lie algebra of all infinitesimal