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On the Gauss map of a complete minimal surface in R^m

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§ 1. Introduction.

Let $x: M \rightarrow R^m$ be a (connected, oriented) minimal surface immersed in R^m ($m \geqq 3$). We may consider M as a Riemann surface by associating a holomorphic local coordinate $z=u+iv$ with each positive isothermal local coordinates u, v. We denote by G the (generalized) Gauss map of M, which is a map of M into $P^{m-1}(C)$ defined by $G=\pi \cdot(\partial x / \partial \bar{z})$, where π is the canonical projection of $C^m-\{0\}$ onto $P^{m-1}(C)$. It is well-known that the map $f=\bar{G}$, the conjugate of G, is holomorphic and the image $f(M)$ is contained in the complex quadric $Q_{m-2}(C)$ in $P^{m-1}(C)$ (cf., [7], p. 110). Note that, when $m=3$, we may identify $Q_1(C)$ with the Riemann sphere and the map f may be regarded as a meromorphic function on M.

In [9], R. Osserman showed that the Gauss map of a complete non-flat minimal surface in R^3 cannot omit a set of positive logarithmic capacity in $Q_1(C)$. Subsequently, in [3], S. S. Chern and R. Osserman proved that the Gauss map of a complete minimal surface M of finite total curvature can fail to intersect at most $(m-1)(m+2)/2$ hyperplanes in general position if it is non-degenerate. Moreover, they showed that the Gauss map of a non-flat complete minimal surface in R^m intersects a dense set of hyperplanes. Recently, in [14], F. Xavier obtained a remarkable result that the Gauss map of a complete non-flat minimal surface in R^3 cannot omit 7 points in $Q_1(C)$.

Relating to these results, we shall prove the following theorem in this paper.

MAIN THEOREM. *Let M be a complete minimal surface in R^m. If the Gauss map of M is non-degenerate, it can fail to intersect at most m^2 hyperplanes in general position.*

It is a very interesting problem to obtain the best estimate $q(m)$ ($\leqq m^2$) of the number of hyperplanes having the property in Main Theorem. In the case $m=3$, R. Osserman showed that there exists a non-flat complete minimal surface in R^3 whose Gauss map omits distinct 4 points ([9], p. 72). As its consequence, there exists a complete minimal surface in R^3 whose Gauss map, as a map into

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