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Strongly regular mappings with ANR fibers and shape

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1. Introduction.

In [7], we defined the fiber shape category FR_B which is shape theoretic category analogous to the fiber homotopy category and studied the category FR_B. In this paper, we study some properties of strongly regular mappings with ANR fibers in FR_B. We first prove the following.

(i) Let E, B be compacta and $\dim B<\infty$. If $p: E\to B$ is a strongly regular mapping with ANR fibers, then for any map $q: Y\to B$ of compacta there is a natural bijection $\Phi: [Y, E]_{q,p}\to\langle Y, E\rangle_{q,p}$, where $[Y, E]_{q,p}$ denotes the set of fiber homotopy classes of fiber maps from q to p and $\langle Y, E\rangle_{q,p}$ the set of morphisms from q to p in FR_B.

In [5], S. Ferry proved that if $f: E\to B$ is a strongly regular mapping onto a complete finite dimensional space B and $f^{-1}(b)$ is an ANR for each $b\in B$, then f is a Hurewicz fibration. If $f: E\to B$ is a Hurewicz fibration between compact ANR, then f is a shape fibration. Note that there are Hurewicz fibrations between compacta which are not shape fibrations (e.g. [11, p. 641]). Next, we prove the following.

(ii) Let E, B be compacta and $\dim B<\infty$. If $p: E\to B$ is a strongly regular mapping with ANR fibers, then p is a shape fibration.

As an application of (i) and (ii), we show the following.

(iii) Let E, E' and B be compacta and $\dim B<\infty$. Suppose that $p: E\to B$ and $p': E'\to B$ are strongly regular mappings with ANR fibers. If a fiber map $f: E\to E'$ from p to p' induces a strong shape equivalence, then f is a fiber homotopy equivalence.

2. Definitions.

Throughout this paper, all spaces are metric spaces and maps are continuous functions. We mean by I the unit interval $[0, 1]$ and by Q the Hilbert cube $\prod_{i=1}^{\infty}[-1, 1]$. A map $p: E\to B$ is a *strongly regular mapping* ([1], [5]) if it is a proper map and for each $b_0\in B$ and $\varepsilon>0$ there is a neighborhood U of b_0 in B such that if $b\in U$, then there exist maps $g: p^{-1}(b)\to p^{-1}(b_0)$ and $h: p^{-1}(b_0)\to p^{-1}(b)$