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On some asymptotic properties of systems of entire functions of smooth growth

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1. Introduction.

Let $f=(f_0, f_1, \cdots, f_n)$ $(n\geqq 1)$ be a transcendental system in $|z|<\infty$. That is to say, $f_0, f_1, \cdots, f_n$ are entire functions without common zero and the characteristic function of f defined by H. Cartan ([**3**]):

$$T(r, f)=\frac{1}{2\pi}\int_0^{2\pi} U(re^{i\theta})\, d\theta - U(0),$$

where

$$U(re^{i\theta})=\max_{0\leq j\leq n}\log|f_j(re^{i\theta})|,$$

satisfies the condition

$$\lim_{r\to\infty}\frac{T(r, f)}{\log r}=\infty.$$

Let X be a set of linear combinations $(\not\equiv 0)$ of $f_0, f_1, \cdots, f_n$ with coefficients in C in general position; that is, for any $n+1$ elements

$$a_{0j}f_0+a_{1j}f_1+\cdots+a_{nj}f_n \quad (j=1, \cdots, n+1)$$

in X, $n+1$ vectors $(a_{0j}, a_{1j}, \cdots, a_{nj})$ are linearly independent.

In this paper, we shall give some necessary or sufficient conditions for f to satisfy

$$T(r, f)\sim T(2r, f), \tag{1}$$

where "$A(r)\sim B(r)$" means $\lim_{r\to\infty} A(r)/B(r)=1$, and discuss the relations between the Nevanlinna deficiency of F in X and the asymptotic behaviour of f satisfying (1).

We use the standard notation of the Nevanlinna theory (see [**5**]).

2. Cases of meromorphic functions; some lemmas and problems.

In this section, we shall pick up important results concerning transcendental meromorphic functions g in $|z|<\infty$ which satisfy