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On the perturbation of linear operators in Banach and Hilbert spaces

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Introduction.

This paper is concerned with the stability theory for several properties of linear operators in Banach and Hilbert spaces.

Let A be a linear operator with domain $D(A)$ and range $R(A)$ in a Banach space X. Let B be a linear operator in X, with $D(B)\supset D(A)$. Assume that

(i) there are constants a_0, $b_0\geqq 0$ such that for all $u\in D(A)$,

$$\|Bu\|\leqq a_0\|u\|+b_0\|Au\| . \tag{0.1}$$

In the perturbation theory it is frequently assumed that

(ii) b_0 is less than one.

In fact, under these conditions the following three facts, for example, are well known:

(P1) $A+B$ is closed if and only if A is closed;

(P2) if A is m-accretive, with $D(A)$ dense in X, and B is accretive then $A+B$ is also m-accretive, i.e., if $-A$ is the generator of a contraction semigroup on X then so is $-(A+B)$, too;

(P3) if A is selfadjoint and B is symmetric then $A+B$ is also selfadjoint (when X is a Hilbert space).

The main purpose of this paper is to show that condition (ii) can be replaced by (indeed generalized to)

(iii) for every $u\in D(A)$ there is $g\in F(Au)$ such that

$$\mathrm{Re}\,(Bu,\ g)\geqq -c\|u\|^2-a\|Au\|\|u\|-b\|Au\|^2 ,$$

where a, b $(b<1)$ and c are nonnegative constants.

The appearance of the duality map F on X to its adjoint X^* may be somewhat unfamiliar in the theory of linear operators. But, we need only elementary properties of the duality map. In this connection, we denote by

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