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On construction of Siegel modular forms of degree two

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Introduction. Let κ be an odd positive integer, N a positive integer divisible by 4, and χ a character modulo N. We denote by $\mathfrak{S}_{\kappa}(N, \chi)$ the space of modular cusp forms of Neben-type χ and of weight $\kappa/2$ with respect to $\Gamma_0(N)$ and denote by $T_{\kappa,\chi}^{N}(p^2)$ the Hecke operator defined on $\mathfrak{S}_{\kappa}(N, \chi)$. We denote by $S_k^{(2)}(L, \psi)$ the space of Siegel modular cusp forms of Neben-type ψ and of weight k with respect to $\Gamma_0^{(2)}(L)=\left\{\begin{pmatrix} A & B \\ C & D \end{pmatrix} \in Sp(2, \boldsymbol{Z}) \,\middle|\, C\equiv 0 \pmod{L}\right\}$. Let $T_{k,\psi}^{(2),L}(n)$ denote the Hecke operator on $S_k^{(2)}(L, \psi)$.

In this paper we discuss two problems. The first problem is a construction of Siegel modular forms of degree two from modular cusp forms of half integral weight. The second one is a construction of modular cusp forms of half integral weight from Siegel modular forms of degree two.

In §1 we show the existence of a linear mapping $\Psi_k^{M,\chi}: \mathfrak{S}_{2k-1}(\tilde{M}, \chi) \to S_k^{(2)}(M, \bar{\chi})$ where M and k are even positive integers, $\tilde{M}=\text{l. c. m.}(4, M)$ and χ is a character modulo M. In §2, using the same method as in [3], we determine Fourier coefficients of $\Psi_k^{M,\chi}(f)$ at infinity. In §3 we study a relation between Andrianov's zeta function associated with $\Psi_k^{M,\chi}(f)$ and Shimura's one associated with f, where $f\in\mathfrak{S}_{2k-1}(\tilde{M}, \chi)$. In [3], we have treated the case $M=2$.

In §4 we give a linear mapping $I_k(L, \psi): \mathcal{M}_k^{(2)}(L, \psi) \to \mathfrak{S}_{2k-1}(\tilde{L}, \psi)$ which is a generalization of the mapping given in [4] and [5], where $4\nmid L$, $\tilde{L}=\text{l. c. m.}(4, L)$ and $\mathcal{M}_k^{(2)}(L, \psi)$ denotes the Maaß's space of $S_k^{(2)}(L, \psi)$.

In the last section we present an application of the results in §1, §2, §3 and §4. *With some assumption on M we show the existence of an isomorphic mapping $\tilde{\Psi}_k^{M,\chi}$ of $\tilde{\mathfrak{S}}_{2k-1}(\tilde{M}, \chi)$ onto $\tilde{\mathcal{M}}_k^{(2)}(M, \chi)$ with the following properties: if $f\in\tilde{\mathfrak{S}}_{2k-1}(\tilde{M}, \chi)$ satisfies $T_{2k-1,\chi}^{\tilde{M}}(p^2)f=\omega_p f$ for every prime p, then $\tilde{\Psi}_k^{M,\chi}(f)$ satisfies $T_{k,\chi}^{(2),M}(n)(\tilde{\Psi}_k^{M,\chi}(f))=\tilde{\lambda}(n)(\tilde{\Psi}_k^{M,\chi}(f))$ for every positive integer n and moreover,*

$$L(2s-2k+4, \chi^2) \sum_{n=1}^{\infty} \tilde{\lambda}(n) n^{-s}$$

$$=L(s-k+1, \chi)L(s-k+2, \chi)\prod_{p}(1-\omega_p p^{-s}+\chi(p)^2 p^{2k-3-2s})^{-1},$$