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Impossibility criterion of being an ample divisor*)

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In [**So 1**], Sommese gave many examples of manifolds that cannot be ample divisors in any manifold. His theory works also to construct non-smoothable singularities (see [**So 2**]). In this note we give the following criterion:

THEOREM. *Let A be a manifold such that $H^1(A, T[-L])=0$ for any ample line bundle L on A, where T is the tangent bundle of A. Then A cannot be an ample divisor in any manifold unless $A \cong \boldsymbol{P}^n$.*

As we shall see in § 1, this result follows easily from a characterization theorem of projective spaces due to Mori-Sumihiro [**MS**]. In § 2, we show that various types of manifolds, including many of those in [**So 1**], satisfy the above criterion. In § 3, similarly as in [**So 2**], we construct examples of non-smoothable singularities.

Notation, convention and terminology.

Usually we employ the notation which is commonly used in algebraic geometry. We work in the category of $\boldsymbol{C}$-schemes of finite type. In most cases everything is assumed to be proper over Spec($\boldsymbol{C}$). *Point* means a closed point. *Variety* is an irreducible, reduced scheme. *Manifold* is a non-singular variety. Vector bundles are confused with locally free sheaves. Line bundles are regarded as linear equivalence classes of divisors, and their tensor products are denoted additively.

Now we list up some symbols.

$[D]$: The line bundle associated with a (Cartier) divisor D.

$\mathcal{F}[L]$: $\mathcal{F} \otimes_{\mathcal{O}} \mathcal{L}$, where $\mathcal{F}$ is a coherent sheaf and $\mathcal{L}$ is the invertible sheaf corresponding to a line bundle L.

T^M: The tangent bundle of a manifold M.

E_X: The pull back of a vector bundle E on Y by a morphism $X \to Y$. Sometimes we write simply E instead of E_X, when there is no danger of confusion.

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