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Invariant subspaces of shift operators of arbitrary multiplicity

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Introduction.

Let $\mathfrak{K}$ be a separable complex Hilbert space and $\{\mathfrak{K}_n\}_{n\in Z}$ be the countable family of copies of $\mathfrak{K}$, where Z is the set of all integers. Let $\mathfrak{H}$ be the direct sum $\sum_{n\in Z}\oplus\mathfrak{K}_n$ of $\{\mathfrak{K}_n\}_{n\in Z}$. A unitary operator U on $\mathfrak{H}$ is called a *shift* operator if U maps $\mathfrak{K}_n$ onto $\mathfrak{K}_{n+1}$ for all n in Z. We denote by S the shift operator on $\mathfrak{H}$:

$$x=\sum_{n\in Z}\oplus\xi_n \longrightarrow y=\sum_{n\in Z}\oplus\eta_n\,,$$

where $\eta_n=\xi_{n-1}$.

A study of invariant subspaces of the shift operator S was originated by the Beurling's paper [2], in which he completely described the structure of invariant subspaces of the unilateral shift operator of multiplicity one. The so-called Beurling's theorem for invariant subspaces was stated in Helson [4] and Halmos [3]. They demonstrated the theorem from geometric consideration in contrast to a function theoretic proof by Beurling. Halmos, in his paper [3], considered the case of countable (and finite) multiplicity and noticed that the study of shift-invariant subspaces might be useful for the case of general bounded linear operators on a Hilbert space. The works [6], [7] of McAsey, Muhly and Saitô were the first attempt to characterize invariant subspaces of a family of shift operators. But they seem to study non-self-adjoint algebras rather than invariant subspaces. Thus the underlying Hilbert space in their paper heavily depends on the structure of algebras.

Our purpose is to study the structure of invariant subspaces of a family $\mathcal{S}$ of shift operators on a given Hilbert space. In our discussion, we use geometric methods in the theory of operator algebras as in [6] and [7]. In the present paper, we give a necessary and sufficient condition for a family $\mathcal{S}$ under which every invariant subspaces is of Beurling type. The condition, of course, is

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