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The Schur index over the 2-adic field

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Let k be a field of characteristic 0 and let B be a cyclotomic algebra over k; that is, a crossed product $(\alpha, k(\zeta)/k)$ in which ζ is a root of unity and α is a factor set on $\mathrm{Gal}(k(\zeta)/k)$ having only roots of unity as values. R. Brauer [**1**], [**2**] and E. Witt [**5**] reduced the problem of determining the Schur index of a character of a finite group to the case of handling the index of a cyclotomic algebra. And E. Witt [**5**] gave a formula of index of it which is central over the rational p-adic field Q_p. But in order to investigate the Schur index and the Schur group of an algebraic number field in detail, it is necessary to obtain the formula of index of a cyclotomic algebra which is central over an arbitrary extension k of Q_p. And this was done by the author [**6**, Theorem 3] for an odd prime p (for the application of the formula, see [**3**], [**6**], [**9**], [**10**]). In [**9**, Theorem 5.6] we also handled the remaining case $p=2$ and obtained a formula when the field k and the factor set α satisfy some conditions.

The purpose of the paper is to settle the case $p=2$ completely. Namely, for any finite extension k of the rational 2-adic field Q_2, we give the formula of index of any cyclotomic algebra $(\alpha, k(\zeta)/k)$ which is central over k (Theorem 2). This will be achieved by embedding the field $k(\zeta)$ into a field L, where the residue class degree of L is sufficiently large and a primitive 2^m-th root of unity ζ_{2^m} belongs to L with a sufficiently large integer m. Thus using this formula for the 2-adic field Q_2 and the formula for the p-adic field Q_p $(p\neq 2)$ in [**6**, Theorem 3], combined with the Brauer-Witt theorem [**9**, p. 31], we can determine the Schur index of a character of a finite group, over an algebraic number field.

NOTATION. For a finite extension field K of the 2-adic numbers Q_2, $\varepsilon_2(K)$ (resp. $\varepsilon'(K)$) is the group of roots of unity whose orders are of 2-power order (resp. relatively prime to 2). For a natural number m, ζ_m is a primitive m-th root of unity. If L is a Galois extension of K then $\mathcal{G}(L/K)$ is the Galois group of L over K. $|\mathcal{G}(L/K)|$ is the order of $\mathcal{G}(L/K)$. $|\sigma|$ is the order of $\sigma\in\mathcal{G}(L/K)$. $\mathcal{T}(L/K)$ is the inertia group of the extension L/K. $e(L/K)=|\mathcal{T}(L/K)|=$the ramification index of L/K. If M is a Galois extension of K such that $M\supset L\supset K$, then for $\sigma\in\mathcal{G}(M/K)$, $\sigma|L$ is the restriction of σ on L. If ζ is a root of unity any subfield of $Q_2(\zeta)$ is called a cyclotomic extension of Q_2.