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Complexes and L-structures

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§0. Introduction.

The purpose of this paper is to study the simplicial complex K with Whitehead topology from the point of view of L-structures. It will be shown that the capacity of K to admit L-structures decreases as the dimension of K increases. As a consequence we know that there is a gap between the class of M_1-spaces and the class of weak L-spaces. Throughout the paper K is a simplicial complex with Whitehead topology and simplexes of K are so-called open ones. K^n denotes the n-section of K. As for terminology refer to the first author [**3**], [**4**] and [**5**].

§1. K with $\dim K \leqq 2$.

1.1. THEOREM. *If* $\dim K \leqq 1$, *then* K *is an* L-*space.*

PROOF. When $\dim K \leqq 0$, K is discrete and metrizable. Consider the case when $\dim K=1$. Let H be an arbitrary closed set of K. Let $\{s_\alpha : \alpha \in A\}$ be the set of 1-simplexes of K. Let U be an open set of K with

$$K^0-H \subset U \subset \bar{U} \subset K-H.$$

For each $\alpha \in A$, let $\mathcal{U}_\alpha$ be an approaching anti-cover of $(H \cap \bar{s}_\alpha) \cup \partial s_\alpha$ in $\bar{s}_\alpha$. Set

$$\mathcal{U}=(\cup \{\mathcal{U}_\alpha : \alpha \in A\}) \cup \{U\}.$$

Then $\mathcal{U}$ is as can easily be seen an approaching anti-cover of H in K. That completes the proof.

1.2. THEOREM. *Let* K *be the* 2-*section of an infinite full complex. Then* K *is not an* L-*space.*

PROOF. Let s be a 1-simplex of K and $\{s_i : i=1, 2, \cdots\}$ a sequence of distinct 2-simplexes of K having s as their common face. Let p be an edge point of s and $\{p_i\}$ a sequence of points of s with $\lim p_i = p$. Let $\mathcal{U}$ be an arbitrary anti-cover of $\{p\}$. Choose $U_i \in \mathcal{U}$ with $p_i \in U_i$. Since $U_i \cap s_i \neq \emptyset$ for any i, we can pick a point $q_i \in U_i \cap s_i$ for each i. Set

$$Z=\{q_i : i=1, 2, \cdots\}.$$