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Vector bundles on ample divisors

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Introduction.

Suppose that a scheme A lies as an ample divisor in another scheme M. Then, as we saw in [5] and [2], the structure of M is closely related to that of A. Keeping this principle in mind, we study in § 1 the behaviour of a vector bundle F on M in relation to that of F_A. In § 2 and § 3 we prove the following extendability criterion announced in [1]: *A vector bundle E can be extended to a vector bundle on M if $H^2(A, \mathcal{E}nd(E)[-tA])=0$ for any $t \geqq 1$, $H^p(A, E[tA])=0$ for any $0<p<\dim A$, $t \in \boldsymbol{Z}$ and if M is non-singular.* In § 4 and § 5, as an application, we show that the Grassmann variety $G_{n,r}$ parametrizing r-dimensional linear subspaces of an n-dimensional vector space cannot be an ample divisor in any manifold except the well known classical cases, namely the cases in which $r=1$, $r=n-1$ or $(n, r)=(4, 2)$.

Notation, Convention and Terminology.

In this paper we fix once for all an algebraically closed field k of any characteristic and assume that everything is defined over k. Basically we employ the same notation as in [2]. In particular, vector bundles are confused with the locally free sheaves of their sections. Here we show examples of symbols.

$E^{\vee}$: The dual vector bundle of a vector bundle E.

S^iE: The i-th symmetric product bundle of E.

$\mathcal{E}nd(E)$: $=\mathcal{H}om(E, E)=E^{\vee}\otimes E$.

$\mathcal{F}[E]$: $=\mathcal{F}\otimes_{\mathcal{O}}\mathcal{O}[E]$ where F is a coherent $\mathcal{O}$-module.

$[D]$: The line bundle associated with a Cartier divisor D.

$Bs\Lambda$: The intersection of all the members of a linear system Λ.

Note that a line bundle L is generated by its global sections if and only if $Bs|L|=\emptyset$.

ρ_{Λ}: The rational mapping induced by Λ.

L_T: The pull back of L to T.