

## Compact Lie group actions and fiber homotopy type

By Katsuo KAWAKUBO

(Received Dec. 16, 1978)

(Revised July 12, 1979)

### 1. Introduction.

In [2], Atiyah proved that the stable fiber homotopy type of the tangent sphere bundle of a differentiable manifold  $M$  depends only on the homotopy type of  $M$ . This theorem not only is interesting in itself, but also plays an important role in constructing and classifying homotopy equivalent manifolds by making use of the Browder-Novikov theory [4], [18], [22], [9], [12].

The purpose of the present paper is to exploit fundamental tools in equivariant topology, and to prove an equivariant analogue of Atiyah's theorem as an immediate consequence.

Throughout the paper,  $G$  will denote a compact Lie group.

We shall first develop general theory on equivariant deformation retracts. For this, we give a filtration on a compact  $G$ -manifold and introduce a concept of  *$G$ -deformation retracts preserving the filtration*. Here we do not assume Yang's results [25], because a gap in the proof was found. Hence we proceed without using [10], [16].

Although Dold's theorem mod  $k$  for  $k > 1$  in the sense of Adams [1] has no counterpart in equivariant theory in general, we shall establish fortunately equivariant Dold's theorem which corresponds to  $k=1$ . Here we have introduced the notion of a *nice  $G$ -deformation retract*, which enables us to carry out the program of the inductive proof of equivariant Dold's theorem one step further.

By combining these results and those of Rubinsztein [20], we shall verify our main theorem.

Needless to say, the theorem can be applied to constructing and classifying  $G$ -homotopy equivalent manifolds just as in the ordinary non-equivariant case.

As an application peculiar to the realm of equivariant topology, we shall have, for instance, that the normal representations of the corresponding components of the fixed point sets of  $G$ -homotopy equivalent manifolds are stably homotopy equivalent in the sense of tom Dieck [6]. On the other hand, examples will be provided to show that the normal representations of the corre-