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Differential equations associated with elliptic surfaces

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Introduction.

The purpose of this paper is to study a certain class of algebraic differential equations which arises in conjunction with the study of elliptic surfaces. The results utilize the general theory of elliptic surfaces due to Kodaira [**11**] and [**12**].

Let E be an elliptic surface over a base curve X. We denote by $\mathcal{J}$ and G the functional and homological invariants of E/X. On a Zariski open subset $X_0 \subset X$, G can be viewed as either a locally constant $\boldsymbol{Z} \oplus \boldsymbol{Z}$ sheaf or as representation $\pi_1(X_0) \to SL_2(\boldsymbol{Z})$. This representation corresponds to an algebraic vector bundle of rank two on X together with an integrable algebraic connection having regular singular points (Deligne [**2**], Griffiths [**3**]), which is known as the Gauss-Manin connection (Katz and Oda [**10**]). It can also be expressed as a second order algebraic differential equation on X having regular singular points. The aim of this research is to make explicit which algebraic differential equations on X arise from elliptic surfaces in this manner and to investigate certain geometric and arithmetic properties of elliptic surfaces by making use of the differential equations with their monodromy representations and conversely. Central to this investigation is the determination of when two elliptic surfaces over X give rise to equivalent homological invariants (representations) and/or when two of the differential equations have equivalent monodromy and therefore give rise to the same flat vector bundle on X.

In Part I we recall some of the notions that we will be dealing with and by direct calculation obtain some information about the differential equations. In Part II we change our viewpoint. Poincaré [**16**] posed the problem of determining the monodromy representation of a given algebraic differential equation on a curve X. That problem remains unsolved. Instead we determine all differential equations with $SL_2(\boldsymbol{Z})$-monodromy and positivity. This is done without reference to elliptic surfaces, but we are able to see in Part III that these equations are precisely those arising from elliptic surfaces. The remainder of

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