J. Math. Soc. Japan
Vol. 33, No. 1, 1981

Approximation of certain classes of periodic functions with many variables

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(Received Aug. 23, 1978)
(Revised March 19, 1979)

§ 0. Introduction.

Let Π_n be the class of all trigonometric polynomials of degree n or less. If f is a continuous 2π-periodic function, the n-th degree of approximation for f is defined by

$$E_n(f)=\inf_{T\in\Pi_n}\|f-T\|=\inf_{T\in\Pi_n}\sup_{|x|\leq\pi}|f(x)-T(x)|.$$

Let the class $W^{(p)}$ ($p\geqq 1$) consist of all the 2π-periodic functions for which there exists a $(p-1)$-th absolutely continuous derivative $f^{(p-1)}(x)$, and $|f^{(p)}(x)|\leqq 1$ almost everywhere. The exact value of $E_n(W^{(p)})=\sup_{f\in W^{(p)}}E_n(f)$ is well-known.

THEOREM A. (Favard [1], Akhiezer and Krein [2]) *The degree of approximation of the classes* $W^{(p)}$, $p=1, 2, \cdots$ *is given by*

$$E_{n-1}(W^{(p)})=K_p n^{-p}, \qquad n=1, 2, \cdots,$$

where

$$K_p=(4/\pi)\sum_{k=0}^{\infty}(-1)^{k(p+1)}(2k+1)^{-p-1}. \tag{0.1}$$

The class $\tilde{W}^{(p)}$ conjugate to $W^{(p)}$ consists of all conjugate functions $\tilde{f}$ of $f\in W^{(p)}$, that is,

$$\tilde{W}^{(p)}=\{\tilde{f}\,;\tilde{f}(x)=(-2/\pi)\int_0^{\pi}[f(x+t)-f(x-t)]\cot(t/2)dt,\ f\in W^{(p)}\}.$$

The exact value of $E_n(\tilde{W}^{(p)})$ is also known.

THEOREM B. (Akhiezer and Krein [2]) *The degree of approximation of the classes* $\tilde{W}^{(p)}$, $p=1, 2, \cdots$ *is given by*

$$E_{n-1}(\tilde{W}^{(p)})=\tilde{K}_p n^{-p}, \qquad n=1, 2, \cdots,$$

where

$$\tilde{K}_p=(4/\pi)\sum_{k=0}^{\infty}(-1)^{kp}(2k+1)^{-p-1}. \tag{0.2}$$