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Non existence of irreducible birecurrent Riemannian manifold of dimension $\geqq 3$

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Introduction.

Formerly, A. Lichnerowicz [1] defined a *birecurrent* (recurrent of the 2nd order) Riemannian manifold by $\nabla^2 R = R \otimes a$, where R is the Riemannian curvature tensor field, a is a covariant tensor field of order 2 and ∇ is the covariant differential. He proved that if a birecurrent M is compact and the scalar curvature does nowhere vanish it is recurrent in the ordinary sense: $\nabla R = R \otimes \alpha$, where α is a 1-form on M. W. Roter [2] treated this problem, but it contains some errors.

It is known (Kobayashi-Nomizu [3], p. 305) that an irreducible recurrent Riemannian manifold of dimension n is locally symmetric if $n \geqq 3$ and whether it is irreducible or not, the universal covering manifold $\tilde{M}$ of a connected complete recurrent Riemannian M is either a globally symmetric space or $M = R^{n-2} \times V^2$, where R^{n-2} is an $(n-2)$-dimensional flat manifold and V^2 is a 2-dimensional Riemannian manifold. The main purpose of this paper is to prove the following theorem.

THEOREM. *If an irreducible Riemannian manifold M of dimension n $(\geqq 3)$ is birecurrent, then M is recurrent in the ordinary sense.*

The case where $n=2$ or M is reducible will be also considered in § 3.

§ 1. Preliminary lemmas.

Although the following discussions are available for Riemannian manifolds of class C^4, we suppose the manifolds to be of class C^∞ for simplicity. 'Differentiable' always means 'C^∞-differentiable'. We use the local expression of each tensor field with respect to a local coordinate system $(x^1, \cdots, x^n)$. The indices run from 1 to n and the summation convention is adopted. The Riemannian metric of M is denoted by g whose components are (g_{ij}) or (g^{ij}). The components of curvature tensor field R are given by