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The modified analytic trivialization of singularities

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We consider the classification of certain classes of singularities. They include in particular those of Kuiper, Whitney and Zeeman. The kite singularities, such as $y^2=t^2x^3+x^5$ in $\boldsymbol{R}^3$, which arise from the Ratio Test ((4)), are also considered.

While facing a classification problem, it is often very difficult, and yet most interesting, to decide which equivalence relation is the best. It should be as strong as possible, whilst keeping the number of classes to a minimum.

A typical situation is reflected in the Whitney example

$$W(x, y; t)=y(y-x)(y-2x)(y-tx), \quad 2<t<\infty .$$

This is considered as a t-parameter family of function germs in $\boldsymbol{R}^2$. Since the contour maps of W, for fixed values of t, have the "same type", these function germs should be put in one equivalence class.

It is easy to see that there exists a t-level preserving homeomorphism $h: \boldsymbol{R}^2\times I\rightarrow\boldsymbol{R}^2\times I$, $h(0, t)=(0, t)$, $I=[a, b]\subset(2, \infty)$, for which $W\circ h$ is independent of t. We then ask whether it is possible to find an h which is C^r-diffeomorphic, or even bianalytic.

Accordingly, we call: homeo$\rightarrow C^1$-diffeo$\rightarrow\cdots\rightarrow C^\infty$-diffeo$\rightarrow$bianaly the canonical route of advance. An equivalence relation by a homeomorphism preserves only the topology, it is too weak for analysis; that by a C^r-diffeomorphism preserves some formality of analysis, but not computability. A bianalytic equivalence, whilst desirable, rarely exists.

In 1965, Whitney pointed out that for his example, no local C^1-diffeomorphism could exist! Thus one can not edge forward at all along the canonical route.

We introduce the notion of *modified analytic trivialization* (MAT). The associated equivalence relation preserves computability, but is slightly weaker than bianalyticity; it is independent of C^r-diffeomorphism ($1\leqq r\leqq\infty$), and much stronger than homeomorphism. This yields an alternative route of advance.

The General Theorem in §3 establishes MAT for a class of singularities in $\boldsymbol{R}^n$. Trivializations for the Kuiper, Whitney and Zeeman singularities are special cases (Theorems 1 to 3). On the other hand, in §4, we find that the kite