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Approximation problem restricted by an incidence matrix

By Ryozi SAKAI

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§ 0. Introduction.

A matrix $E=(e_{ij})_{j=0,\cdots,s}^{i=1,\cdots,k}$ is called an incidence matrix if $e_{ij}=0$ or 1. Let $e=\{(i, j)\,;\, e_{ij}=1\}$ and $|e|=\sum e_{ij}$. In this paper we consider both the "algebraic case" and the "trigonometric case", simultaneously. Thus, through this paper we assume that $s=\max\{j\,;\,(i, j)\in e\}$ and that

$$\bar{e}=\begin{cases} |e|-1 & \text{in the algebraic case,} \\ [(|e|-1)/2] & \text{in the trigonometric case,} \end{cases}$$

where $[x]$ is the largest integer such that $[x]\leqq x$. Let Π_n denote the algebraic or trigonometric polynomials of degree n or less. Let A denote an interval $[0, 1]$ or unit circle $K=[-\pi, \pi)$. Given k distinct points $x_1, \cdots, x_k\in A$ and a polynomial $P\in\Pi_{\bar{e}}$. If $P^{(j)}(x_i)=0$ for $(i, j)\in e$ implies $P=0$, we said that the scheme $S=(E\,;\,\{x_i\})$ is poised. If the scheme S is poised for all choices of nodes $\{x_i\}$, E is called a poised matrix. In the algebraic case, a wide class of poised matrices has been found. In order to mention them, we need several definitions. Given an incidence matrix E, we define

$$m_j=\sum_{i=1}^{k} e_{ij} \quad\text{and}\quad M_p=\sum_{j=0}^{p} m_j\,, \qquad j,\ p=0, \cdots, s\,.$$

An incidence matrix E is said to satisfy the Pólya conditions if

$$M_p\geqq p+1\,, \qquad p=0, \cdots, s\,. \tag{0.1}$$

A sequence of 1's in a row of E;

$$e_{ij}=e_{ij+1}=\cdots=e_{ij+r-1}=1\,, \tag{0.2}$$

is called a block if its length r is maximum. A block is even or odd according as its length r is even or odd. A block (0.2) is called a Hermite block if $j=0$.

THEOREM 0.1. (Ferguson [1], Atkinson and Sharma [2]) *In the algebraic polynomial class $\Pi_{\bar{e}}$, an incidence matrix satisfying* (0.1) *is poised if its interior rows contain no odd blocks of non Hermite data.*