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On extensions of Lie algebras by algebras

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Introduction.

Let k be a commutative ring of prime characteristic p. Let A be a k-algebra, and let L be a restricted Lie algebra (p-Lie algebra) over k. We assume A and L are finitely generated projective k-modules. The first aim of this article is to establish a categorical correspondence among the following three kinds of objects;

I) Pairs of an extension of affine k-group schemes

$$1 \longrightarrow \mu^{A} \longrightarrow G \longrightarrow \mathcal{E}(L) \longrightarrow 1$$

and an admissible homomorphism

$$\rho: G \longrightarrow \mathrm{Aut}(A),$$

where μ^{A} and $\mathrm{Aut}(A)$ denote the k-group functors which send each commutative k-algebra T to

$$T \mapsto \text{the group of units in } T \otimes A,$$

$$T \mapsto \text{the group of } T\text{-algebra automorphisms of } T \otimes A,$$

respectively, and $\mathcal{E}(L)$ denotes the finite k-group scheme associated with L [**2**, p. 277]. By ρ admissible we mean that

$$gxg^{-1} = \rho(g)(x), \quad \forall g \in G(T), \quad \forall x \in \mu^{A}(T),$$

$$xyx^{-1} = \rho(x)(y), \quad \forall x \in \mu^{A}(T), \quad \forall y \in T \otimes A,$$

for each commutative k-algebra T.

II) Exact sequences of restricted Lie algebras

$$0 \longrightarrow A \longrightarrow X \longrightarrow L \longrightarrow 0$$

such that for each $x \in X$,

$$\mathrm{ad}(x) = [x, -]: A \longrightarrow A$$

is a k-algebra derivation.