J. Math. Soc. Japan
Vol. 32, No. 2, 1980

Global and local equivariant characteristic numbers of G-manifolds

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(Received June 29, 1978)

§1. Introduction and statement of results.

Let G be a compact Lie group and $h_G(\ )$ be an equivariant multiplicative cohomology theory. Let M and N be closed G-manifolds of class C^3. Then for a G-map $f: M \to N$, we defined an "equivariant Gysin homomorphism"

$$f_!: h_G(M) \longrightarrow h_G(N)$$

under certain conditions and obtained equivariant Riemann-Roch type theorems in general [**13**], [**14**]. When N is a point, $f_!$ is called an "index homomorphism" and is denoted by Ind. On the other hand, we got a localization theorem. Consequently by virtue of the functorial property of our equivariant Gysin homomorphism, we have many equations between invariants of a G-manifold and fixed point data.

In the present paper, we shall confine ourselves to two special cases. Let $G \to EG \to BG$ be the universal principal G-bundle.

Case 1. $G = T^n$ (torus), $h_G(M) = H^*(EG \underset{G}{\times} M : R)$ where R is the real number field, manifolds are oriented G-manifolds of class C^3.

Case 2. $G = (Z_2)^n$, $h_G(M) = H^*(EG \underset{G}{\times} M ; Z_2)$, manifolds are non oriented G-manifolds of class C^3,

The greater part of the results in Case 1 will be those in [**12**]. The results in Case 2 will be analogous to those in Case 1 and include the main theorems of [**17**], [**18**].

First we shall show that our $f_!$ has the functorial property and is an $h_G(*)$-module homomorphism where $*$ stands for a point. Now we consider the set $S \subset h_G(*)$ of Euler classes of the vector bundles $EG \underset{\phi}{\times} R^m \to BG$ where G acts on R^m by representations $\phi: G \to O(m)$ without trivial direct summand. Then S is a multiplicative set of $h_G(*)$. It follows that we get a localization $S^{-1}h_G(M)$ and an induced homomorphism

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Research supported in part by Fūjukai grant.