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On covariant representations of continuous C^*-dynamical systems

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Introduction.

The study of C^*-dynamical systems plays an important role in the theory of C^*-algebras. This paper is devoted to a study of covariant representations of continuous C^*-dynamical systems. A C^*-dynamical system is a pair (A, G), where A is a C^*-algebra and G is a locally compact Hausdorff group acting on A by *-automorphisms. The action of $g\in G$ on $a\in A$ is denoted by $g\cdot a$ or ga. If, for all $a\in A$, the map $g\mapsto g\cdot a$ of G into A is continuous for the norm topology of A, we say that the C^*-dynamical system (A, G) is continuous. From a continuous dynamical system (A, G), one can construct the crossed product $C^*(G, A)$, the covariance algebra in the sense of [6]. For a closed subgroup G_0 of G, there is a method to construct representations of $C^*(G, A)$ from covariant representations of (A, G_0), which are called the induced representations ([**10**], § 3). On the other hand, in [8], W. Krieger showed the construction of a von Neumann algebra from a commutative dynamical system $((M, \mathfrak{B}, m), G)$, where $(M, \mathfrak{B}, m)$ is a measure space and G is a countable discrete group. This construction coincides with that of the crossed product when the action of G is free.

In this paper, to study the continuous C^*-dynamical system (A, G), we try to apply the idea of Krieger's to the covariant representations of (A, G). For this purpose, in Section 1, we show the construction of covariant representations of (A, G) from representations of A, which is an analogue of the Krieger's construction, and then we construct a representation $\mathrm{Cent}\,\rho$ of $C^*(G, A)$ from a representation ρ of A. If the action of G on the quasi-dual $\hat{A}$ of A is free, $\mathrm{Cent}\,\rho$ coincides with the induced representation of ρ. Using the representation $\mathrm{Cent}\,\rho$, we show the construction of a C^*-algebra G^*A from (A, G), which is different from that of the crossed product. In Section 2, we show that, if representations ρ_1 and ρ_2 of A are quasi-equivalent, then $\mathrm{Cent}\,\rho_1$ and $\mathrm{Cent}\,\rho_2$ are quasi-equivalent.

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