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On characteristic classes of conformal and projective foliations

Dedicated to Professor A. Komatu on his 70th birthday

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0. Introduction.

In this paper, we define characteristic classes for conformal and projective foliations and investigate the relationship of them with those for smooth foliations defined by Bott and Haefliger [4] and those for Riemannian foliations due to Lazarov and Pasternack [16] and Kamber and Tondeur [12] (see also [18]). For a construction of the characteristic classes of smooth foliations [2], Bott's vanishing theorem [1] concerning the Pontrjagin classes of the normal bundles played an important role. Also Pasternack's vanishing theorem for the Riemannian foliations [25] was the starting point of Lazarov-Pasternack theory. Similarly our motivation for the present work was the strong vanishing theorem of Nishikawa and Sato [22], which states that the ring generated by the Pontrjagin classes of the normal bundle of a conformal or projective foliation is trivial for cohomology degree>codimension. However we do not use this theorem in our construction. Instead, we follow the Bott-Haefliger approach [4] to the characteristic classes of smooth foliations (namely, à la Gelfand-Fuks theory——see [3]), and also the method of Kamber and Tondeur used in their theory of characteristic classes for foliated bundles [12] [13]. Thus just as the cohomology of some truncated Weil algebra of $\mathfrak{gl}(n; \boldsymbol{R})$ or $\mathfrak{so}(n)$ played the role of characteristic classes for smooth or Riemannian foliations, our characteristic classes also take the form of the cohomology of certain truncated Weil algebra of $\mathfrak{so}(n+1, 1)$ for the conformal case and of $\mathfrak{sl}(n+1; \boldsymbol{R})$ for the projective case, where $\mathfrak{gl}(n; \boldsymbol{R})$, $\mathfrak{so}(n)$, $\mathfrak{so}(n+1, 1)$ and $\mathfrak{sl}(n+1; \boldsymbol{R})$ are the Lie algebras of $GL(n; \boldsymbol{R})$, $SO(n)$, $SO(n+1, 1)$ and $PGL(n; \boldsymbol{R})$ respectively. The main point of our construction is the use of Cartan connection, by which we have also shown that there are other characteristic classes for Riemannian foliations which are not covered by Lazarov-

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