J. Math. Soc. Japan
Vol. 31, No. 4, 1979

On a theorem for linear evolution equations of hyperbolic type

By Kazuo KOBAYASI

(Received Jan. 18, 1978)
(Revised Oct. 19, 1978)

0. Introduction.

In [1] and [2] T. Kato gave some fundamental and important theorems about evolution operator associated with linear evolution equations

$$du/dt+A(t)u=f(t)\,, \qquad 0\leqq t\leqq T\,,$$

of "*hyperbolic*" type in a Banach space X. Here, f is a given function from $[0, T]$ into X, $A(t)$ is a given linear operator which is a negative generator of a C_0-semigroup in X, and the unknown function u is from $[0, T]$ into X. Those theorems are useful in applications to symmetric hyperbolic systems of partial differential equations (for example, see [3] and [7]). The proofs were carried out by using a device due to Yosida [8, 9], and the proof of Theorem 6.1 of [1] was simplified later by Dorroh [4]. It is assumed in those articles that $A(t)$ is norm continuous from $[0, T]$ into $B(Y, X)$, where Y is a Banach space densely and continuously embedded in X. However, we find it useful to strengthen the theorems by replacing the norm continuity of $A(t)$ with strong continuity. The purpose of the present paper is to show that Theorem 6.1 of [1] is still valid if we assume the strong continuity of $A(t)$ instead of the norm continuity of it. In Section 1 our result is stated. In Section 2 we give a proof of it. In this paper we refer to [1] for notations and definitions.

The author would like to express his hearty thanks to Professor T. Kato for his valuable advice through his letter.

1. Statement of Theorem.

Let X and Y be Banach spaces such that Y is densely and continuously embedded in X. We denote by $\|\ \|$ and $\|\ \|_Y$ norms of X and Y, respectively, and by $B(Y, X)$ the set of all bounded linear operators on Y to X. The operator norm of $A\in B(Y, X)$ is denoted by $\|A\|_{Y,X}$. We write $B(X)$ for $B(X, X)$ and $\|A\|$ for $\|A\|_{X,X}$. Let $\{A(t)\}$ be a family of linear operators in X, defined for $t\in I=[0, T]$, such that $-A(t)$ is the infinitesimal generator of