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Rational homotopy type and self maps

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§ 1. Introduction.

D. Sullivan showed in [1] and [2] that the rational homotopy type of a simply connected simplicial complex can be algebraically described by its minimal model which is constructed from the Q-polynomial forms on it. If the minimal model of a simplicial complex K is isomorphic to the one obtained from its cohomology ring, the rational homotopy type of K is called a formal consequence of the cohomology ring. Complexes, having such a rational homotopy type, enjoy interesting homotopy properties ([2]). The purpose of this paper is to characterize such complexes by the existence of a certain kind of self maps. Since a finite CW-complex has the same homotopy type as a polyhedron we work in the category of simply connected finite CW-complexes. Our main result is

THEOREM. *Let K be a simply connected finite CW-complex. Then the following three conditions on K are equivalent.*

(1) *The rational homotopy type of K is a formal consequence of the cohomology ring.*

(2) *For any integer r, there exists a multiple s of r and a map $f: K\rightarrow K$ such that $f^*=s^*Id: H^*(K;Z)\rightarrow H^*(K;Z)$.*

(3) *There exists a rational number t ($t\neq 0, \pm 1$) and a map $F: K_{(0)}\rightarrow K_{(0)}$ such that $F^*=t^*Id: H^*(K_{(0)};Z)\rightarrow H^*(K_{(0)};Z)$, where $K_{(0)}$ denotes the localization of K at zero and the homomorphism s^*Id denotes the homomorphism s^iId for each degree i.*

From this Theorem, we can deduce

COROLLARY 1. *If the rational homotopy type of K is a formal consequence of the cohomology ring then K is 0-universal.*

COROLLARY 2. *Let K be a simply connected finite CW-complex. Then there exists a simply connected finite CW-complex $\tilde{K}$ satisfying the following conditions:*

(1) *$H^*(K;Q)=H^*(\tilde{K};Q)$ as a ring,*

(2) *$\tilde{K}$ is 0-universal.*

This paper is organized as follows:

In § 2 we give a brief account of minimal models and recall two Sullivan's