J. Math. Soc. Japan
Vol. 31, No. 1, 1979

Banach algebra structure in Fourier spaces and generalization of harmonic analysis on locally compact groups

By Masayuki FUJITA

(Received March 28, 1977)

Abstract.

Let (M, G, α) be a continuous W^*-dynamical system. Then, the predual $(G \otimes_\alpha M)_*$ of the crossed product $G \otimes_\alpha M$ of M by G can be turned into a Banach algebra and some of the notions and theorems in harmonic analysis on locally compact groups are extended to the corresponding ones in the crossed products. Among others, one can get a criterion for T in $G \otimes_\alpha M$ to fall in M: $T=\pi_\alpha(x)$ for some $x \neq 0$ in M if and only if the support of T reduces to the unit e in G.

1. Introduction.

Generalizing the so-called Pontryagin's duality theorem for locally compact abelian groups, K. Saito [6] proved that, for a general locally compact group G, the predual $\mathfrak{M}(G)_*$ of $\mathfrak{M}(G)$, the von Neumann algebra generated by the left regular representation of G, becomes an involutive commutative Banach algebra by suitably introducing the multiplication in it and the spectrum space $\widehat{\mathfrak{M}(G)}_*$ of $\mathfrak{M}(G)_*$ is homeomorphic to the original group G.

P. Eymard [3], on the other hand, regarded $\mathfrak{M}(G)_*$ as a regular function algebra $A(G)$ on G. He called it the Fourier algebra of G and he showed, by using techniques in function algebras and in von Neumann algebras, that some of the notions and results of harmonic analysis on locally compact abelian groups can be extended to the non-abelian case.

Recently, in order to have an explicit form of the element of a continuous W^*-crossed product $G \otimes_\alpha M$, H. Takai [8] introduced the notion of Fourier spaces and, under the condition that (M, G, α) be a G-finite separable continuous W^*-dynamical system, he showed that Gelfand-Raikov's and Godement's theorems in harmonic analysis on locally compact groups can be generalized in (M, G, α). And he added a remark that the predual $(G \otimes_\alpha M)_*$ can be regarded as the space of continuous functions of G into the predual M_* and