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On the values at rational integers of the p-adic Dirichlet L functions

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Introduction.

Let χ be a primitive Dirichlet character with conductor f_χ, and $L(s, \chi)$ be the Dirichlet L series associated with χ. For any prime number p, we denote by $L_p(s, \chi)$ the p-adic L function introduced by Kubota and Leopoldt [1]. We fix an embedding ι of the algebraic closure $\bar{\boldsymbol{Q}}(\subset \boldsymbol{C})$ into $\bar{\boldsymbol{Q}}_p$ once for all. By this ι, we identify any formal power series $\sum_{n=0}^{\infty} a_n X^n \in \bar{\boldsymbol{Q}}[[X]] \subset \boldsymbol{C}[[X]]$ (resp. any number $a \in \bar{\boldsymbol{Q}}$) with $\sum_{n=0}^{\infty} \iota(a_n) X^n \in \bar{\boldsymbol{Q}}_p[[X]]$ (resp. $\iota(a) \in \bar{\boldsymbol{Q}}_p$). We assume that all the Dirichlet characters we consider are primitive.

In this paper, firstly, we present a formula for the values of $L_p(s, \chi)$ at positive integers. To simplify the description of the main result, we assume that p is an odd prime number (for the case of $p=2$, see Theorem A of this paper) and that f_χ is neither 1 nor p. For a fixed prime number p, let ω be the Dirichlet character defined by $\omega(x) \equiv x \bmod p$ for $x \in \boldsymbol{Z}$. We set $\chi_j = \chi \cdot \omega^{-j}$ and

$$B(X, j) = \frac{\tau(\chi_j)}{f_{\chi_j}} \sum_{m=1}^{f_{\chi_j}} \bar{\chi}_j(m) \log\left(1 + \frac{X}{1-\exp(2\pi\sqrt{-1}\, m/f_{\chi_j})}\right) \in \bar{\boldsymbol{Q}}_p[[X]],$$

$$\left(\text{viz. } \frac{\tau(\chi_j)}{f_{\chi_j}} \sum_{m=1}^{f_{\chi_j}} \bar{\chi}_j(m) \sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k} \left(\frac{X}{1-\exp(2\pi\sqrt{-1}\, m/f_{\chi_j})}\right)^k\right)$$

for $j = 0, 1, 2, \cdots, p-2$,

where f_{χ_j} and $\tau(\chi_j)$ denote the conductor of χ_j and the Gaussian sum of $\chi_j \left(= \sum_{t=1}^{f_{\chi_j}} \chi_j(t) \exp(2\pi\sqrt{-1}\, t/f_{\chi_j})\right)$ respectively. Further we denote by S the formal integral operator acting on a certain subspace Q_K of formal power series with coefficients in a finite extesion field K of $\boldsymbol{Q}_p$, given by