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# On the fundamental group of the complement of certain plane curves

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## § 0. Notations.

Throughout this paper, we use the following notations.

$\boldsymbol{Z}$: the integers or an infinite cyclic group

$(n_1, n_2, \cdots, n_k)$: the greatest common divisor of $n_1, n_2, \cdots, n_k$

$G, G_1, G_2$: groups

$Z(G)$: the center of $G$

$D(G)$: the commutator group of $G$

$G_1 * G_2, G_1 * G_2 * G$: the free product of $G_1$ and $G_2$ or of $G_1, G_2$ and $G$ respectively

$\boldsymbol{Z}_p$: a cyclic group of order $p$

$F(p)$: a free group of rank $p$

$\{e\}$: the trivial group

$e$: the unit element

$\left.\begin{array}{l} G(p\,;q) \\ G(p\,;q\,;r) \end{array}\right\}$: special groups. See the definitions in § 2.

## § 1. Introduction and statement of results.

Let $C$ be an irreducible curve in the projective space $\boldsymbol{P}^2$ and let $G$ be the fundamental group of the complement of $C$. So far known, we have only two cases: (I) $G$ is infinite and the commutator group $D(G)$ is a free group of a finite rank (Zariski [8]; Oka [6]). (II) $G$ is a finite group (Zariski [8]).

We do not know whether this is true or not in general. The purpose of this paper is to give a theorem which says that, for a certain case, we have only the case (I). Namely let

$$C: \prod_{j=1}^{l} (Y-\beta_j Z)^{\nu_j} - \prod_{i=1}^{m} (X-\alpha_i Z)^{\lambda_i} = 0 \tag{1.1}$$

where $X$, $Y$ and $Z$ are homogenous coordinates of $\boldsymbol{P}^2$ and