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Tôki covering surfaces and their applications

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An infinite and unbounded covering surface $R^{\sim}$ of an open Riemann surface R is referred to as a *Tôki covering surface* if any bounded harmonic function on $R^{\sim}$ is constant on $\pi^{-1}(q)$ for each q in R where π is the projection. The primary purpose of this paper is to show the existence of a Tôki covering surface $R^{\sim}$ of *any* given open Riemann surface R (Main theorem in no. 1.2). We can construct $R^{\sim}$ so that the projections of branch points in $R^{\sim}$ is discrete in R. Remove a parametric disk V from R. We will show that any bounded harmonic function on $R^{\sim}-\pi^{-1}(\bar{V})$ vanishing on its boundary relative to $R^{\sim}$ is constant on $\pi^{-1}(q)$ for each q in $R-\bar{V}$, and actually we will prove this assertion for a more general subset than V (Theorem in no. 2.5). As an application of this we will see that $\pi^{-1}(V)$ always clusters to the Royden harmonic boundary of $R^{\sim}$ which consists of a single point (Theorem in no. 2.3). Based on these results we will show that there exists a single point of positive harmonic measure but no isolated point in the Royden harmonic boundary of $R^{\sim}-\pi^{-1}(\bar{V})$ (Theorem in no. 3.1). The most effective application of Tôki covering surfaces is the following: For any compact Stonean space Δ which is a Wiener harmonic boundary of a hyperbolic Riemann surface, there exists an open Riemann surface whose Royden harmonic boundary consists of a single point and whose Wiener harmonic boundary is Δ (Theorem in no. 4.3). We denote by $b(W)$ (the B-harmonic dimension) the number of isolated points in the Wiener harmonic boundary of an open Riemann surface W and by $d(W)$ (the D-harmonic dimension) and $d^{\sim}(W)$ (the $D^{\sim}$-harmonic dimension) the numbers of isolated points and points with positive harmonic measures, respectively, in the Royden harmonic boundary of W. Based on the above results we will determine the triples $(b, d, d^{\sim})$ of countable cardinal numbers such that $(b, d, d^{\sim})=(b(W), d(W), d^{\sim}(W))$ for a certain open Riemann surface W (Theorem in no. 5.3).

Tôki covering surfaces.

1.1. We start by fixing terminologies. Let $R^{\sim}$ and R be Riemann surfaces. The triple $(R^{\sim}, R, \pi)$ is said to be a *covering surface* if $\pi: R^{\sim}\to R$ is a non-constant analytic mapping. The surface R is referred to as the *base surface*