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Addendum to my paper "On Veronese manifolds"

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In the present paper we use the same definitions and notations as ones in [1]. Moreover, using the inequalities in [1] and [2], we give an interesting characterization of Veronese manifolds as follows;

THEOREM. *Let M be an n-dimensional compact orientable submanifold which is minimally immersed in an $(n+p)$-dimensional sphere of constant curvature $\tilde{c}$. If the immersion is full and the sectional curvatures of M are not smaller than $\frac{n\tilde{c}}{2(n+1)}$, then M is a sphere of constant curvature $\tilde{c}$ or M is a Veronese manifold.*

PROOF. Since M is compact, let c be the minimum of all sectional curvatures of M. In this case, in [2], S. T. Yau gave the following inequality:

$$\sum h_{ij}^{\alpha} \Delta h_{ij}^{\alpha} \geqq n(a+1)cS - a\tilde{c}nS + aL_N - \frac{1-a}{2} K_N \,, \tag{1}$$

where a is a constant such that $a+1 \geqq 0$. Using the inequality (2.1) $K_N \leqq nL_N$ in [1], from (1) we get

$$\sum h_{ij}^{\alpha} \Delta h_{ij}^{\alpha} \geqq (a+1)cnS - a\tilde{c}nS + \left(\frac{a}{n} - \frac{1-a}{2}\right) K_N \,, \tag{2}$$

where a is a positive constant.

Now, setting $a = \frac{n}{n+2}$ in (2), we have the inequality

$$\sum h_{ij}^{\alpha} \Delta h_{ij}^{\alpha} \geqq \frac{2n(n+1)}{n+2} \left\{ c - \frac{n}{2(n+1)} \tilde{c} \right\} S \,. \tag{3}$$

On the other hand, by definition we know $\frac{1}{2} \Delta S = \sum (h_{ijk}^{\alpha})^2 + \sum h_{ij}^{\alpha} \Delta h_{ij}^{\alpha}$. This equality, together with (3) and our assumption, implies

$$\frac{1}{2} \Delta S \geqq \sum (h_{ijk}^{\alpha})^2 + \frac{2n(n+1)}{n+2} \left\{ c - \frac{n}{2(n+1)} \tilde{c} \right\} S \geqq 0 \,. \tag{4}$$

Since M is compact and orientable, $\Delta S = 0$ on M. Then all equalities in (1), (2), (3) and (4) hold everywhere on M, that is, the immersion is isotropic,