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Scattering theory for Schrödinger equations with potentials periodic in time

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§ 1. Introduction.

Let $\mathcal{H}$ be the Hilbert space $L^2(\boldsymbol{R}^n)(n\geqq 3)$ and let $\{H(t)=H_0+V(t),\ t\in\boldsymbol{R}^1\}$ be a family of Schrödinger operators in $\mathcal{H}$ with a time-dependent perturbation $V(t)$, where $H_0=-\Delta$ is the negative Laplacian in $\boldsymbol{R}^n$. We suppose that $\{-iH(t)\,;\ t\in\boldsymbol{R}^1\}$ generates a unitary evolution group $\{U(t, s)\,;\ -\infty<t, s<\infty\}$. Fundamental problems in scattering theory under these circumstances are as follows. (1) When do the strong limits

$$W_{\pm}(s)f=\lim_{t\to\pm\infty}U(t, s)^{-1}e^{-i(t-s)H_0}f$$

exist for every $f\in\mathcal{H}$ and every $s\in\boldsymbol{R}^1$? (2) If the above limits exist, how can we characterize their ranges $R(W_{\pm}(s))$, in particular, do their ranges coincide (completeness of wave operators)?

The study of the problems has begun in recent years. However, most works appeared so far are concerned with the problems under the assumption that the perturbation $V(t)$ vanishes sufficiently rapidly as $|t|\to\infty$ so that $W_{\pm}(s)$ turn out to be unitary. In the case that the perturbation $V(t)$ is periodic in time, on the other hand, the wave operators $W_{\pm}(s)$ are not unitary in general. This case was first taken up by Schmidt [**19**] who proved, among other things, the existence and the completenese of $W_{\pm}(s)$, determining their ranges precisely. In [**19**] this result was proved in the situation that $V(t)$ is an operator of trace class for each t; and for $V(t)$ given by a potential $v(t, x)$, it was conjectured that if $v(t, \cdot)\in L^2(\boldsymbol{R}^3)\cap L^1(\boldsymbol{R}^3)$ the existence and the completeness of $W_{\pm}(s)$ would hold.

The purpose of the present paper is to study the problems in the case that the perturbation $V(t)$ is periodic in time and given by a time-dependent potential $v(t, x)$. We shall prove that the conjecture of Schmidt holds good under weaker conditions. Namely, we first assume the following Assumption (A.1).

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