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The formal system for various 3-valued logics II

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§ 1. Introduction and semantics.

We gave the Gentzen-type formal system of Kleene's 3-valued logic and McCarthy's 3-valued logic interpreted into the system in [3]. In this paper we shall give the Gentzen-type formal system of McCarthy's logic itself. After that, we shall give the formal system in which McCarthy's and Kleene's are joined. In this system, serial or parallel evaluation is mixed.

We shall use the same terminology in [3]. Especially we use the symbols $+$, $\cdot$ or $\supset_M$ for *'or'*, *'and'* or *'implies'* in McCarthy's sense respectively and $\vee$, $\wedge$ or $\supset$ for them in Kleene's sense respectively, and use $\neg$ for *'not'* in common.

As we see from the following truth tables, formulas are evaluated serially from left to right in McCarthy's logic while in parallel in Kleene's logic.

$A+B$

$A\backslash B$	t	ω	f
t	t	t	t
ω	ω	ω	ω
f	t	ω	f

$A\cdot B$

$A\backslash B$	t	ω	f
t	t	ω	f
ω	ω	ω	ω
f	f	f	f

$A\supset_M B$

$A\backslash B$	t	ω	f
t	t	ω	f
ω	ω	ω	ω
f	t	t	t

$A\vee B$

$A\backslash B$	t	ω	f
t	t	t	t
ω	t	ω	ω
f	t	ω	f

$A\wedge B$

$A\backslash B$	t	ω	f
t	t	ω	f
ω	ω	ω	f
f	f	f	f

$A\supset B$

$A\backslash B$	t	ω	f
t	t	ω	f
ω	t	ω	ω
f	t	t	t

A	$\neg A$
t	f
ω	ω
f	t

Here t, f or ω means *'true'*, *'false'* or *'undefined'* respectively.

It is clear that $P_1\cdot P_2\cdot\cdots\cdot P_n$ (or $P_1+P_2+\cdots+P_n$) has the value t (or f) if