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On the growing up problem for semilinear heat equations

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§ 1. Introduction.

We will consider the Cauchy problem for the semilinear heat equation

$$\frac{\partial u}{\partial t}=\Delta u+f(u), \qquad t>0,\ x\in R^{d}, \tag{1.1}$$

with the initial condition $u(0, x)=a(x)$. It is assumed that the function f is defined, non-negative and locally Lipschitz continuous in $[0, \infty)$. If the initial value $a(x)$ is a bounded non-negative continuous function in R^d, not vanishing identically, then it is well-known that there exists a positive local solution $u(t, x)$ of (1.1); more precisely, there exist positive T $(\leqq\infty)$ and $u(t, x)$ satisfying the following conditions (i), (ii) and (iii).

(i) $u(t, x)$ is defined on $[0, T)\times R^d$, strictly positive in $(0, T)\times R^d$ and $u(0, x)=a(x)$.

(ii) For any $T'<T$, $u(t, x)$ is bounded and continuous on $[0, T']\times R^d$.

(iii) $\frac{\partial u}{\partial t}$ and $\frac{\partial^2 u}{\partial x_i \partial x_j}$ $(1\leqq i, j\leqq d)$ exist in $(0, T)\times R^d$ and $u(t, x)$ satisfies (1.1) in the classical sense.

If $T_\infty=T_\infty(a, f)$ denotes the supremum of all T satisfying the above three conditions, then the existence of *global solution* is the case $T_\infty=\infty$, and in the general situation $(T_\infty\leqq\infty)$ the unique existence assertion amounts to say that there exists a unique solution $u(t, x)$ of (1.1) up to T_∞ satisfying the above three conditions with $T=T_\infty$. In this paper, such a solution is called simply a *positive solution* of (1.1), and is denoted by $u(t, x; a, f)$ when we want to elucidate the initial value $a(x)$ and the nonlinear term $f(u)$. A positive solution of (1.1) is said to *blow up in a finite time* and the corresponding T_∞ is called *the blowing-up time* of the solution, provided that $T_\infty<\infty$. A global positive solution $u(t, x)$ of (1.1) is said to *grow up to infinity,* if for each positive constant M and each compact set K in R^d there exists $T<\infty$ such that $t>T$ and $x\in K$ imply $u(t, x)>M$.

The purpose of this paper is to investigate the following problem: How