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On the singular spectrum of boundary values of real analytic solutions

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In this article we give an estimation of the singular spectrum of boundary values of real analytic solutions of linear partial differential equations with constant coefficients. The result has been expected from the study of continuation of real analytic solutions. It gives a unified aspect to many problems on continuation of regular solutions. (See [**5**]. But the present result (Theorem 2.1) is a little weaker than the conjecture in [**5**].) Our estimate is sharp for the wave equations or for the ultrahyperbolic equations (Example 2.5).

Before my work, Professor Komatsu has given a vast conjecture (unpublished) on precise determination of the singular spectrum of the boundary values of general hyperfunction solutions from the standpoint of purely boundary-value-theoretical origin. Corollary 2.4 is a partial answer to his conjecture. Also in Theorem 2.1 we have added the distinction of the signs in the conormal direction taking account of his conjecture. I am very grateful to Professor Komatsu for his kind advices.

In § 1, we paraphrase the definition of boundary values by way of the Fourier transform. It was originally given for the equations with analytic coefficients in [**9**] employing the Cauchy-Kowalevsky theorem. In § 2 we prove the main theorem. In § 3, we give an application to continuation of real analytic solutions. In some sense it is an extension of the results in [**4**] or [**5**].

§ 1. Boundary values of hyperfunction solutions.

First we prepare the notation. $p(D)$ denotes a linear partial differential operator with constant coefficients, where $D=(D_1, \cdots, D_n)$ and $D_i=\sqrt{-1}\,\partial/\partial x_i$, $i=1, \cdots, n$. By the Fourier transform

$$\tilde{u}=\mathcal{F}[u]=\int u(x)e^{\sqrt{-1}x\xi}dx\,, \qquad x\xi=x_1\xi_1+\cdots+x_n\xi_n\,,$$

it corresponds to the multiplication operator $p(\xi)$. ζ denotes the complexifica-

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