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A normalization theorem in formal theories of natural numbers

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In this paper we deal with two formal number theories, i.e. a classical and an intuitionistic one. They are obtained from Gentzen's well-known logical systems LK and LJ by adding the principle of mathematical induction as an inference rule. Our aim is to prove that we can transform any derivation in these systems into its so-called normal form. We shall explain later what a normal derivation is. To put it briefly, it means a proof without redundancy. It will be defined not as the derivation which can not be transformed any more but as the derivation satisfying some conditions on variables and inference rules. In the proof of our assertion we apply the transfinite induction up to ε_0 just as Gentzen did in his second consistency proof of number theory.

Our normalization theorem yields some by-products. Examining structures of normal derivations, we shall obtain the following results: the consistency of number theory, and Harrop's result (a disjunctive and an existential property for intuitionistic number theory). These results are obtained as the direct consequences of our theorem.

There are several investigations on normalization theorems in formal number theories. For example, Jervell [**5**], Martin-Löf [**6**], Prawitz [**7**], Troelstra [**13**] and Zucker [**14**] study them for several systems of natural deduction. And Scarpellini [**8**]-[**11**] obtain some related results for systems of sequent calculus. Later we shall refer to them again.

In §1 we introduce our formal systems. We state our main theorem and show its applications there. In §2 we paraphrase the theorem so that we could simplify our way of thinking for the proof. After some preparations the paraphrased theorem is proved. In these sections we treat both the classical and the intuitionistic system simultaneously. And, if necessary, we give a certain notice to distinguish them.

Most of our concepts are due to Gentzen [**2**]. With a few exceptions we use the same terminology in the same senses as those in the English translation of it.